Prof. Hadley

Three Dollars per Annum, in Advance. Single Copies, Twenty-Five Cents.

Vol. III. No. VIII.

THE

MATHEMATICAL MONTHLY.

MAY, 1861.

EDITED BY

J. D. RUNKLE, A.M., A.A.S.

CAMBRIDGE:
PUBLISHED BY SEVER AND FRANCIS.
NEW YORK:
A. S. BARNES AND BURR.
LONDON: TRÜBNER & CO.
1861.

Encyclopædia Britannica.

EIGHTH EDITION.

Published by LITTLE, BROWN, & CO.,

110 Washington Street, Boston,

THE ENCYCLOPÆDIA BRITANNICA; or, Dictionary of Arts, Sciences, and General Literature. Eighth Edition, revised, enlarged, and brought up to the present time. Edited by THOMAS STUART TRAILL, M.D., F.R.S.E., Professor of Medical Jurisprudence in the University of Edinburgh. With upwards of five hundred Engravings on steel, and many thousands on wood. Complete in 21 vols. 4to. Cloth, $ 115.50.

This edition has undergone careful revision and extensive alterations, so as to accommodate it to the improved taste and advanced intelligence of the times. The editor has secured the co-operation of the most eminent living authors, who have contributed treatises in the various departments of Science, Literature, the Arts, Manufactures, Commerce, Statistics, and General Knowledge, to supersede those now rendered obsolete by the progress of discovery, improvements in the arts, or the general advancement of society.

Among the contributors to this edition are: Right Hon. T. B. Macaulay, Right Rev. Richard Whately, William Whewell, D.D., Sir William Hamilton, Sir Archibald Alison, Isaac Taylor, Rev. Charles Kingsley, Prof. J. D. Forbes, Robert Stephenson, Richard Monckton Milnes, Hepworth Dixon, Theodore Martin, Prof. J. Y. Simpson, Thomas de Quincey, R. G. Latham, W. E. Aytoun, Sir David Brewster, J. R. M'Culloch, Dr. Doran, Lawrence Oliphant, Prof. J. S. Blackie, James Montgomery, Dr. William Gregory, John Hill Burton, Alexander Smith, Rev. William Hanna, LL.D., Leonhard Schmitz, LL.D., Charles Tomlinson.

☞ A complete Index to the work is in press, and will be issued shortly.

THE MATHEMATICAL MONTHLY.

The excellent notices which the Monthly continues to receive from both sides of the Atlantic must be a source of great encouragement to all its friends. The *Lady's and Gentleman's Diary* for 1861, edited by the eminent mathematician, PROF. W. S. B. WOOLHOUSE, contains the following: —

"This American periodical, which has now completed its second volume, continues to be most ably conducted, and is well supplied by talented correspondents. Besides numerous mathematical questions and solutions, it comprises original papers of considerable variety and sterling interest, and its publication must contribute largely to the diffusion and advancement of mathematical science."

A recent number of the *Boston Daily Transcript* gives the following item: —

"AMERICAN LITERATURE ABROAD. — Fourteen columns of the last Athenæum are devoted to two American authors, — Motley and Emerson. That paper, which rarely commends anything from this side of the Atlantic, gives 'The Mathematical Monthly' the following first-rate notice. That magazine is published at Cambridge, and edited by Mr. J. D Runkle: —

"'This is a collection of simple things for comparative elementary students. Our English mathematical journals are very lofty, and the beginner has no part in them. This American journal is full of pleasing elementary matter, and contains some things worthy the attention of the finished mathematician. We should like to see the same sort of thing in England; but where are we to find the editor? He must be of high knowledge and elementary turn combined, with firmness enough to keep his journal down to the intelligible point by refusal of contributions of too lofty a character, and skill enough to keep it up to the interesting point by selecting from the loads of lower learning with which he would be inundated.'"

Those wishing India-proof impressions of SIR JOHN HERSCHEL'S or PROF. PEIRCE'S Portrait, will please address the Publishers.

Those in arrears are earnestly requested to remit their subscriptions, if possible, before the issue of the next number.

SEVER & FRANCIS, PUBLISHERS,
Cambridge, Mass.

Contents.

MAY, 1861.

	PAGE
PRIZE PROBLEMS FOR STUDENTS	229
AWARD OF THE PRIZES FOR SOLUTIONS OF PROBLEMS IN No. IV., VOL. III.	230
ON THE GEOMETRICAL CONSTRUCTION OF CERTAIN CURVES BY POINTS. By Prof. H. A. Newton	235
SECOND PRIZE ESSAY — ON CENTRAL AND DISTURBING FORCES. By David Trowbridge	245
NOTE ON A QUESTION IN INTEREST. By Prof. J. C. Porter	254
NOTE ON THE GENERAL THEORY OF EQUATIONS. By Prof. E. W. Evans	256
NOTE ON TRIGONOMETRICAL FORMULAS. By Prof. R. M. Moore	258
MATHEMATICAL MONTHLY NOTICES	259

Terms to Subscribers and Advertisers.

No. of Copies to one Address.	Volume.	Terms.	Premium to Clubs.	Advertisement Pages.	First Insertion.	Subsequent Insertions.
1	III.	$ 3.00		1	$ 15.00	$ 12.00
2	III.	5.00		½	9.00	7.00
3	I., II., III.	6.00		¼	5.00	4.00
5	III.	11.00	III.	⅛	3.00	2.00
10	III.	20.00	II., III.			
15	III.	30.00	I., II., III.			

Parties in New York wishing to advertise in the Mathematical Monthly will please send their favors to Messrs. A. S. BARNES AND BURR by the 15th of each month. All club subscriptions and business communications must be addressed to

SEVER AND FRANCIS, Publishers,
Cambridge, Mass.

University Press, Cambridge: Electrotyped and Printed by Welch, Bigelow, & Co.

Greenleaf's Course of Mathematics,

FOR ALL GRADES OF SCHOLARS,

IN SIX BOOKS.

FOR COMMON SCHOOLS.

1. New Primary Arithmetic.
2. Intellectual Arithmetic.
3. Common School Arithmetic.

HIGH SCHOOLS AND ACADEMIES.

4. National Arithmetic.
5. Treatise on Algebra.
6. Elements of Geometry.

The Author and Publishers of this series have spared no labor and expense to adapt the books to meet the demands of the present day, — with its improved methods of instruction

The series, in its revised and improved form, surpasses all others, —

1. *In the lucid, progressive, and strictly philosophical and scientific* ARRANGEMENT *of the several parts and subjects.*

2. *In the clearness and conciseness of the* DEFINITIONS *and* RULES, *and in the completeness of the* ILLUSTRATIONS.

3. *In the variety and practical character of the* PROBLEMS ; *which, from their nature, tend to interest the pupil, exercise his ingenuity, and secure useful mental discipline.*

4. *In being a* CONSECUTIVE SERIES, *graded to the wants of* PRIMARY, INTERMEDIATE, GRAMMAR *and* HIGH SCHOOLS, ACADEMIES, NORMAL SCHOOLS, *and* COMMERCIAL COLLEGES.

Greenleaf's Arithmetics are now widely used throughout the United States, by the best Educators, and the sale of them is greater than that of any other series published ; — an indication that the books possess *enduring merits*

The attention of Teachers of the Higher Schools is invited to the ELEMENTS OF GEOMETRY, recently published It has received the commendation of many eminent Teachers, a few of which are subjoined.

PROF J. G. HOYT, *Chancellor of Washington University, St. Louis,* writes: "After a somewhat careful examination of Greenleaf's Geometry, I have decided to subject it to the test of the Recitation-Room. Of course I should not do this if I were not pretty well satisfied of its excellence as a text-book. It strikes me that there is some improvement upon Legendre in the elementary portions of the work ; but the chief points of attraction to me are the copious and well-considered Problems in the Eleventh, Twelfth, and Thirteenth Books."

From A. M GAY, A. M., *Principal of the High School, Charlestown, Mass.*

"From an examination and a practical test in some of my classes, of the advanced sheets of Greenleaf's Geometry, I have no hesitation in pronouncing it superior to any class-book of the kind with which I am acquainted. The basis of the work is Legendre's Elementary Geometry, which has been long and favorably known to teachers. Much valuable matter, however, has been added by the present editor, so that it cannot be regarded as a translation, or a mere compilation. The language in which the propositions are stated is remarkably clear and concise. The development of principles is sufficiently extended, and in all respects it seems a book well adapted to the purposes of instruction in Academies, High Schools, and other institutions of learning.

From D. B. HAGAR, A. M., *President of the American Inst. of Instruction, etc.*

"Having carefully read Greenleaf's Geometry, the work appears to me to be one of great merit. Of course no person can expect to find much that is essentially *new* in a geometrical treatise published at this day. But Mr. Greenleaf's, while it is based like nearly all American Geometries, upon Legendre's original work, contains valuable propositions not usually found in similar publications, and also presents uncommonly clear demonstrations of many of the standard propositions. The definitions contained in this work are generally concise, clear, and exact. The exercises in Mensuration, and in the Application of Algebra to Geometry, will be found unusually interesting and useful. In a word, I have no doubt that Mr. Greenleaf's Geometry will be found to compare favorably with the best Geometries yet published."

*** All interested are respectfully invited to correspond freely with us, and to examine the revised, permanent, stereotyped editions. OUR DESCRIPTIVE CATALOGUE furnished, on application, by mail.

ROBERT S. DAVIS & CO.,
Publishers, Boston.

VALUABLE SCIENTIFIC WORKS

PUBLISHED BY

LITTLE, BROWN, & CO.,

110 Washington St., Boston.

PEIRCE'S ANALYTICAL MECHANICS.

In One Vol. 4to. Price $7.50.

PHYSICAL AND CELESTIAL MECHANICS : Developed in Four Systems of Analytical Mechanics, Celestial Mechanics, Potential Physics, and Analytical Morphology. By BENJAMIN PEIRCE. Part I. Analytical Mechanics.

"I have re-examined the memoirs of the great geometers, and have striven to consolidate their latest researches and their most exalted forms of thought into a consistent and uniform treatise. If I have, hereby, succeeded in opening to the students of my country a readier access to these choice jewels of intellect ; if their brilliancy is not impaired in this attempt to reset them ; if, in their own constellation, they illustrate each other, and concentrate a stronger light upon the names of their discoverers ; and, still more, if any gem which I may have presumed to add, is not wholly lustreless in the collection, I shall feel that my work has not been in vain." — *Extract from the Preface.*

GAUSS'S THEORIA MOTUS.

In One Vol. 4to. Price $5.

THEORY OF THE MOTIONS OF THE HEAVENLY BODIES MOVING ABOUT THE SUN IN CONIC SECTIONS. A translation of Gauss s Theoria Motus, with an Appendix. By CHARLES HENRY DAVIS.

COOKE'S CHEMICAL PHYSICS.

In One Vol. 8vo. Price $3.

ELEMENTS OF CHEMICAL PHYSICS. By JOSIAH P. COOKE, Jr., Erving Professor of Chemistry and Mineralogy, in Harvard College. Illustrated by 400 Woodcuts. With a copious Index.

The Mathematical Monthly.

IN commencing our third volume, we beg to assure the friends of this Journal, that all legitimate efforts will be made to sustain it, together with the Prizes, which have proved such an excellent feature of its plan. It will be seen (see 2d page of cover) that the amount of the Prizes has been increased. But all efforts on our part must prove unavailing if the subscription list, with other sources of income, will not cover the expenses which we are incurring to make the Monthly really valuable and attractive to all interested in it. To show our friends at home what is thought of the Monthly abroad, we venture to make short extracts from letters lately received

"The work appears to me so excellent, that I have at once ordered the series for the Library of the Observatory.

"G. B. AIRY, *Ast. Royal.*"

"*Royal Obs., Greenwich, July* 5 1860.

"I have looked through it with great interest, and especially because it shows to what an extent interest in mathematical subjects is felt through the United States. I do not believe that it would be possible in England to get up a Mathematical periodical brought out in so handsome a form and supported by so large a number of subscribers.

"*Trinity College, Dublin, July* 12, 1860. GEORGE SALMON."

"DEAR SIR, — I thank you sincerely for the copy you so kindly sent me of the first volume of the Mathematical Monthly. We have not, that I am aware of, any work of a similar character in England. I feel confident it will serve a valuable purpose in stimulating and giving a healthy tone to the study of Mathematics in the higher class of schools, and judging from this volume, it will be a work of as great interest and give as much matter for thought to the tutor as to the student. Original investigations on the mere elementary branches of science are perhaps not often to be expected : those portions have now been so long and so frequently considered, and by so many different minds, that something new — a really original idea — appears only at very distant intervals. Still many interesting questions may, and, as your first volume shows, do arise even in the earliest branches of Mathematical science ; whether discussions on the principles and on the best methods of placing them before the pupil, or examples and problems which in their solutions show a large amount of ingenuity and mathematical skill. The Mathematical Monthly claims a special interest from the very fact of its character rendering it useful to the many without detracting from its scientific merit.

"I remain, your obedient servant.

"*St. John's College, Cambridge, July* 26, 1860. HUGH GODFRAY."

We feel confident that a little effort on the part of our friends would double our subscription list. See terms to clubs and new subscribers taking Vols. I., II., III. We could then engrave some of the fine portraits of LAGRANGE, MONGE, CAUCHY, BIOT, &c., lately received ; and, indeed, incur any expense to increase the value of the Monthly to all Will our friends make up their clubs at once, and notify us, so that we can decide upon the size of our edition ?

GROVER & BAKER'S

Celebrated

SEWING MACHINES,

FOR

Family Use and Manufacturing Purposes.

OVER

40,000

IN DAILY USE

IN THE

United States

AND

Territories,

GIVING

UNIVERSAL

SATISFACTION.

THESE MACHINES sew from two spools; the thread requires no re-winding; and they make a STRONGER, HANDSOMER, AND MORE ELASTIC SEAM than any other Machine. They perform their work with a rapidity never before equalled, are almost noiseless in their operation, and are FAVORITES in every family where they have been introduced.

The "Scientific American," November 5th, 1859, says: —

"The machines manufactured by the Grover & Baker Sewing Machine Company are too well known by the public at large to need any recommendations at our hands, and we will simply add, that we have had one of them in use in our family for some time past, *and it is considered the most useful article in the house, next to the cradle, and no less indispensable than that.*"

"Zion's Herald," Nov. 16th, 1859, says: —

"We would recommend to families and to those about to organize a homestead, *to be sure and have a Family Bible, and also one of Grover & Baker's SEWING MACHINES.*"

GROVER & BAKER SEWING MACHINE CO.,

18 Summer Street, Boston.
495 Broadway, New York.
730 Chestnut Street, Philadelphia.
181 Baltimore Street, Baltimore.
58 West Fourth Street, Cincinnati.
115 Lake Street, Chicago.
124 North Fourth Street, St. Louis.
91 Montgomery Street, San Francisco.

THE

MATHEMATICAL MONTHLY.

Vol. III. . . . MAY, 1861. . . . No. VIII.

PRIZE PROBLEMS FOR STUDENTS.*

I. Let $ABCD$ be a rectangular parallelogram, G, H, two points in AD and DC, and E, F, two points in AB and BC respectively, such that GH equals EF. Join AH, AF, CG, CE, and show that $AH^2 + CG^2 = AF^2 + CE^2$.

II. ABC is an isosceles triangle, AB the base, D a point in a straight line parallel to AB, DE perpendicular to AB; prove that the difference of the triangles ADC, AEC, is a constant area.

III. If in the continued fraction

$$\sqrt{n} = a + \cfrac{1}{b + \cfrac{1}{\cfrac{2c+1}{b+\sqrt{n}}}}$$

we put $a + \frac{1}{b} = M$, $a + \cfrac{1}{b + \frac{1}{c}} = N$, then $MN = n$.

IV. Let

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1, \quad lx + my + nz = 0,$$

be the equations of an ellipse in space; prove that the principal axes of this ellipse are the roots of the equation

$$\frac{l^2 a^2}{z^2 - a^2} + \frac{m^2 b^2}{z^2 - b^2} + \frac{n^2 c^2}{z^2 - c^2} = 0.$$

* The right-hand member of equation 1, Prob. I., March No., should be y^2, instead of $4y^2$.

V. One of two homofocal ellipses is inscribed in a triangle, and the other circumscribed about it; show that the following relation exists between the semi-transverse axes and eccentricity

$$e^8 - 4e^6a^2 + 6e^4a^2a'^2 - 4e^2a^2a'^4 + a^4a'^4 = 0.$$

Solutions of these problems must be received by July 1, 1861.

AWARD OF THE PRIZES FOR SOLUTIONS OF PROBLEMS IN No. IV., Vol. III.*

The first prize is awarded conjointly to G. B. Hicks, Cleveland, Ohio, and David Trowbridge, Perry City, N. Y., their solutions being considered of equal merit.

The second prize is awarded to John Q. Hollister, Hamilton College, Clinton, N. Y.

The third prize is awarded to George W. Pierce, Freshman Class, Harvard College.

The fourth prize is awarded to Marcus M. Rhoades, Mount Pleasant College, Huntsville, Mo.

* To prevent misapprehension, we will state here, that it was never intended that any competitor should draw the same prize twice during the same year, although credit will be given him as being otherwise entitled to it; and no prize will be awarded for solutions which do not in themselves possess sufficient merit to fairly entitle them to it. It is a matter of extreme satisfaction that a few of the competitors always send excellent, and often elegant solutions of all the problems, and are certainly preparing themselves for important and responsible positions demanding mathematical attainments and ability. We sincerely hope to witness a great increase in competition for these prizes, and to this end we now remove the restriction limiting the competition to students in the United States, and invite all students interested in this department of the Monthly to send us their solutions. To the inquiry of our young English friends, we reply, that we shall be most happy to have them compete for and carry off these prizes. Your letters sent to us, *post-paid*, at *Cambridge, Mass., U. S. A.*, will be pretty sure to be promptly received. — Ed.

Prize Solution of Problem I.

By W. W. Lemmon, M'Kendree College, Lebanon, Ill.

$A\,B\,C\,D$ is a square, $A\,C$ its diagonal; bisect $A\,D$ in E, join $B\,E$ cutting $A\,C$ in F; then will

$$\frac{\triangle A\,E\,F}{1} = \frac{\triangle C\,E\,F}{2} = \frac{\triangle A\,B\,E}{3} = \frac{\triangle B\,C\,F}{4}.$$

Bisect $A\,B$ in G and join $F\,G$. It is evident that

$$A\,E\,F = A\,F\,G = G\,F\,B. \quad \therefore A\,E\,F = \tfrac{1}{2}\,A\,F\,B = \tfrac{1}{3}\,A\,E\,B.$$

But $A\,E\,C = A\,E\,B$; subtract $A\,E\,F$ from each, and $C\,E\,F = A\,F\,B$ The triangles $A\,E\,F$, $B\,F\,C$ are similar; therefore

$$\frac{A\,E\,F}{B\,F\,C} = \frac{A\,E^2}{B\,C^2} = \frac{1}{4}. \qquad \therefore\ A\,E\,F = \tfrac{1}{4}\,B\,F\,C.$$

Prize Solution of Problem II.

By several of the Competitors.

If $\dfrac{a-b}{a\,y+b\,x} = \dfrac{b-c}{b\,z+c\,x} = \dfrac{c-a}{c\,y+a\,z} = \dfrac{a+b+c}{a\,x+b\,y+c\,z}$, then each of these fractions $= \dfrac{1}{x+y+z}$.

Since these ratios are equal, any antecedent is to its consequent as the sum of all the antecedents is to the sum of all the consequents; therefore either fraction equals the sum of all the numerators *divided by the sum of all* the denominators,

$$\therefore \frac{a+b+c}{(a+b+c)\,(x+y+z)} = \frac{1}{x+y+z}.$$

Prize Solution of Problem III.

If d be the perpendicular, dropped from the angle C on the opposite side of a triangle, show that $\sin C = \dfrac{c\,d}{a\,b}$. Deduce the formula

$$\sin (A+B) = \sin A \cos B + \cos A \sin B.$$

From the vertex B drop a perpendicular d' on the opposite side. Then $a \sin C = d'$, and calling the area of $A\,B\,C$, A, we get

$$a\,b \sin C = b\,d' = 2\,A = c\,d; \quad \therefore \sin C = \frac{c\,d}{a\,b}.$$

Now, $c = a\cos B + b\cos A$, $d = b\sin A = a\sin B$;

$$\therefore \frac{c\,d}{a\,b} = \sin A\cos B + \cos A\sin B = \sin C = \sin(A+B),$$

since $A+B+C = 180°$. This is substantially the solution given by all the competitors.

PRIZE SOLUTION OF PROBLEM IV.

An hyperbola, having its asymptotes parallel to the axes of co-ordinates, may be made to pass through the angular points of the quadrilateral formed by the four lines whose equations are

(1) $y + x + 6a = 0$, (3) $y + 6x + a = 0$,
(2) $y + 2x + 3a = 0$, (4) $y + 3x + 2a = 0$.

Denote the equations (1), (2), (3), (4) by $\alpha, \beta, \gamma, \delta$; then

$$(1) \qquad \alpha\gamma + k\beta\delta = 0$$

is the equation of a conic passing through the angular points of the quadrilateral $(\alpha\beta\gamma\delta)$; for it is satisfied by $\alpha = 0$, $\beta = 0$; $\alpha = 0$, $\delta = 0$; $\gamma = 0$, $\beta = 0$; $\gamma = 0$, $\delta = 0$; k being an arbitrary constant. We have then

$$(2)\ y^2+7xy+6x^2+7ay+37ax+6a^2+k(y^2+5xy+6x^2+5ay+13ax+6a^2)=0.$$

To remove the second powers of x and y make $k = -1$,

$$(3) \qquad \therefore xy + ay + 12ax = 0.$$

Put $x = x' + b$, $y = y' + c'$, then (3) becomes

$$(4)\quad x'y' + (c+12a)x' + (a+b)y' + bc + ac + 12ab = 0;$$

and by making $c = -12a$, $b = -a$, we get $x'y' = 12a^2$, the well-known equation of the hyperbola referred to its asymptotes. Therefore (3), which passes through the angular points of the quadrilateral, is an hyperbola having its axis of co-ordinates parallel to its asymptotes. This solution was given by Messrs. HICKS and TROWBRIDGE.

Mr. Asher B. Evans gave the following solution: —

Suppose the centre of the hyperbola to be at the point (θ, φ), and remove the origin of coördinates to this point; then will equations, (1), (2), (3), (4) become

(5) $y+x+6a-\theta-\varphi=0$, (7) $y+6x+a-6\theta-\varphi=0$,
(6) $y+2x+3a-2\theta-\varphi=0$, (8) $y+3x+2a-3\theta-\varphi=0$.

Let A, B, C, D represent the intersection of (1) and (2), (1) and (4), (2) and (3), (3) and (4), in forming the quadrilateral $ABCD$. From equations (5), (6), (7), (8), we readily find

$$A=(\theta+3a, \varphi-9a), \qquad B=(\theta+2a, \varphi-8a),$$
$$C=(\theta+\tfrac{1}{2}a, \varphi-4a), \qquad D=(\theta+\tfrac{1}{3}a, \varphi-3a).$$

The equation to the hyperbola, referred to its asymptotes as axes, is

$$(9) \qquad xy=c^2.$$

If the hyperbola pass through A, B, C, we must have

$$(10) \quad xy=c^2=(\theta+3a)(\varphi-9a)=(\theta+2a)(\varphi-8a)$$
$$=(\theta+\tfrac{1}{2}a)(\varphi-4a).$$

Equation (10) gives $\theta=a$, $\varphi=12a$; hence

$$(11) \quad xy=c^2=(\theta+3a)(\varphi-9a)=(\theta+2a)(\varphi-8a)$$
$$=(\theta+\tfrac{1}{2}a)(\varphi-4a)=(\theta+\tfrac{1}{3}a)(\varphi-3a)=12a^2.$$

Since equation (11) exists, the hyperbola which passes through A, B, C, also passes through D, and hence through all the angular points of the quadrilateral $ABCD$.

Prize Solution of Problem V.

By David Trowbridge, Perry City, N. Y.

Three lines, x, y, z, are drawn from the angular points of a triangle to a point within it. Show that when $x^n+y^n+z^n$ is a minimum we have

$$\frac{\sin(x, y)}{z^{n-1}}=\frac{\sin(x, z)}{y^{n-1}}=\frac{\sin(y, z)}{x^{n-1}},$$

in which (x, y) denotes the angle between x and y.

Let ABC be the triangle, and O the point within it, $OC = x$, $OB = y$, $OA = z$, and the angle $(x, a) = \theta$. Then we readily get

$$u = x^n + y^n + z^n,$$

$$= x^n + (a^2 + x^2 - 2ax\cos\theta)^{\frac{n}{2}} + (b^2 + x^2 - 2bx\cos(C-\theta))^{\frac{n}{2}},$$

$$D_x u = nx^{n-1} + ny^{n-2}(x - a\cos\theta) + nz^{n-2}(x - b\cos(C-\theta)) = 0,$$

$$(1) \qquad = x^{n-1} + xy^{n-2} + xz^{n-2} - ay^{n-2}\cos\theta - bz^{n-2}\cos(C-\theta) = 0,$$

$$D_y u = nax y^{n-2}\sin\theta - nbx z^{n-2}\sin(C-\theta) = 0,$$

$$(2) \qquad = ay^{n-2}\sin\theta - bz^{n-2}\sin(C-\theta) = 0.$$

By putting $B = x^{n-2} + y^{n-2} + z^{n-2}$, (1) becomes

$$(3) \qquad Bx = ay^{n-2}\cos\theta + bz^{n-2}\cos(C-\theta).$$

Multiplying (3) by $\sin\theta$, and (2) by $-\cos\theta$, and adding the results, we get

$$Bx\sin\theta = bz^{n-2}\sin C, \qquad \text{or} \qquad Bax\sin\theta = abz^{n-2}\sin C.$$

But $ax\sin\theta = xy\sin(x, y) =$ twice the area of BOC;

$$(4) \quad \therefore \frac{Bxyz\sin(x, y)}{z^{n-1}} = ab\sin C = \text{twice the area of the triangle } ABC.$$

Similarly,

$$\frac{Bxyz\sin(x, z)}{y^{n-1}} = ac\sin B, \qquad \frac{Bxyz\sin(y, z)}{x^{n-1}} = bc\sin A;$$

$$\therefore \frac{\sin(x, y)}{z^{n-1}} = \frac{\sin(x, z)}{y^{n-1}} = \frac{\sin(y, z)}{x^{n-1}}.$$

The condition of a minimum

$$D_x^2 u \,.\, D_y^2 u - (D_{x,y}^2 u)^2 > 0.$$

is readily computed.

Mr. G. B. Hicks computed the values of x, y, z in a very elegant manner for $n = 1$, and $n = 2$, which we reluctantly omit for want of space.

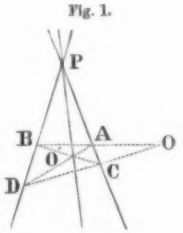
ON THE GEOMETRICAL CONSTRUCTION OF CERTAIN CURVES BY POINTS.

By H. A. Newton, Professor of Mathematics in Yale College.

1. Definition. — *The polar to a poim with respect to an angle* is that straight line which, together with the line drawn from the point to the vertex of the angle, divides the angle harmonically. (*Géom. Sup.*, Art. 347.)*

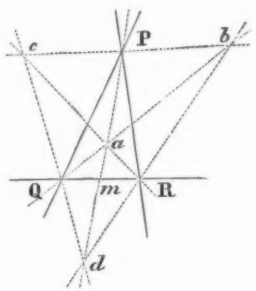
2. It may be constructed by the straight edge alone. For let O (Fig. 1) be the given point, and APB the given angle. Draw OAB and OCD, any two straight lines, through O to cut PA in A and C, and PB in B and D; draw AD and BC to cut each other in O', and draw PO'. The line PO' will be the polar of the point O with respect to the angle APB. (*Géom. Sup.*, Art. 346.)

Fig. 1.



3. Theorem. — Let PQR (Fig. 2) be any triangle, and a any point in the same plane. Draw aP, aQ, aR, and also Pb, Qc, and Rd, the polars to a with respect to the three angles of the triangle PQR. Then the lines Pa, Qc, and Rd will meet in one point, d, the lines Pb, Qa, and Rd will meet in one point, b, and Pb, Qc, and Ra will meet in one point, c.

Fig. 2.



For let Pa meet QR, Qc, and Rd in m, d', and d respectively. The segment Pm is cut harmonically by Qa and Qd', and also by Ra and Rd. Therefore d and d' coincide; or, in other words, Pa, Qc, and Rd meet in one point. In like manner the other lines may

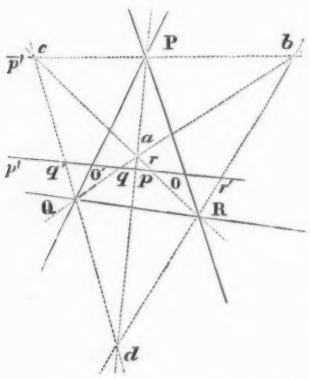
* The references are to the *Traité de Géométrie Supérieure*, par M. Chasles, 8vo, Paris, 1852, Bachelier. We cannot too highly recommend this work of the great French Geometer.

be proved to meet in b, and in c. This proposition is proved also in the *Géom. Sup.*, Art. 373.

4. In the figure thus constructed the six lines joining the four points a, b, c, d may be considered the four sides and two diagonals, or in other words the six sides of a complete quadrilateral, $a\,b\,c\,d$. The points P, Q, and R will be the points of intersection of the three couples of opposite sides.

5. THEOREM. — The three polars of any point, O (Fig. 3), with respect to the three angles formed at P, Q, and R by the opposite sides of the quadrilateral meet in one point, O'.

Fig. 3.



This proposition is demonstrated by CHASLES (*Géom. Sup.*, Art. 347), as follows: Let the polars of O, with respect to the angles $a\,P\,b$ and $a\,Q\,c$, meet in O'; it is to be proved that the line joining O' and R is polar to O, with respect to the angle $a\,R\,d$. Draw $O\,O'$, and let this line meet the sides of the quadrilateral in p, p'; q, q'; r, r'. These six points are in involution (*Géom. Sup.*, Art. 339); and since the segment $O\,O'$ divides harmonically the segments $p\,p'$ and $q\,q'$, the points O and O' are the double points of the involution (*Géom. Sup.*, Art. 205). Hence the segment $r\,r'$ divides the segment $O\,O'$ harmonically, and O' is on the polar of O with respect to the angle $a\,R\,d$.

6. Since there is but one polar to a point with respect to an angle, it is evident that, having constructed the quadrilateral (or having assumed the point a), there is *in general* to one point, O, of the plane $P\,Q\,R$ one point, O', and only one, where the polars of O intersect.

These two points, O and O', have important relations to each other; and in what follows, each will be said to be *conjugate* to the other.

7. If O is on one of the sides of the quadrilateral, as $a\,d$, then,

1st, O' is on the same line $a\,d$; and

2d, the segment, $O\,O'$, divides $a\,d$ harmonically; for,

1st, the polar of O, with respect to the angle $a\,P\,b$, is the side of the angle $P\,a$; and

2d, the lines $O\,R$ and $O'\,R$ must divide the angle $a\,R\,d$, and hence the segment $a\,d$, harmonically.

8. A fasceau of conics circumscribing the quadrilateral $a\,d\,Q\,R$ will, by a well-known theorem, cut the line $b\,c$ in a series of couples of points in involution. And since the two lines $b\,a$, $b\,d$ may be considered as one of these conics, the point b is one of the double points of the involution. The point c, for a like reason, is the other double point. Therefore each segment, as $m\,n$, intercepted on the line $b\,c$ by a conic about $a\,d\,Q\,R$, is divided harmonically by b and c (*Géom. Sup.*, Art. 205). Therefore m and n are points conjugate to each other (Art. 7). Hence, if a conic be described to pass through a and d, two summits of the quadrilateral, and through Q and R, those two angles of the triangle not on the line $a\,d$, then will m and n, the two points where this conic cuts the opposite side, $b\,c$, of the quadrilateral, be conjugate to each other.

9. The point a is entirely arbitrary. The positions of conjugate points may, however, be more readily conceived if a is taken the centre of the circle inscribed in the triangle $P\,Q\,R$. Then will the lines $a\,P$, $a\,Q$, $a\,R$ bisect the angles of the triangle, the lines $P\,b$, $Q\,c$, and $R\,d$ will bisect the exterior angles of the triangle, and be perpendicular respectively to the lines $a\,P$, $a\,Q$, and $a\,R$. The points b, c, and d will be the centres of the circles touching the sides of the triangle $P\,Q\,R$, and the angles $a\,P\,O$, $a\,Q\,O$, and $a\,R\,O$ will be equal respectively to $a\,P\,O'$, $a\,Q\,O'$, and $a\,R\,O'$.

The centre of a circle inscribed in a given triangle cannot, however, be determined by a straight edge alone; and hence, in con-

structions where the compasses are not to be used, the point a must not be subject to the above condition.

10. Definitions. — If the point O move along any curve, its conjugate point O' will describe a second curve. This second curve will be called *the conjugate* of the first curve. The first curve is evidently the conjugate of the second.

The points P, Q, and R will be called *base points*. The three lines $P\,Q$, $P\,R$, and $Q\,R$ will be called *axes*.

11. If two curves intersect in n points, none of which are base points, the two conjugate curves will evidently intersect in n points, none of which are base points.

12. If two curves touch each other at a point not a base point, the two conjugate curves will also touch each other.

For two curves, $\mathfrak{A}$ and $\mathfrak{B}$, are tangent to each other in a point O, when the segment intercepted by them on any transversal indefinitely near to O is an infinitesimal of the second order. Let the straight line $P\,m\,n$, indefinitely near to O, cut $\mathfrak{A}$ and $\mathfrak{B}$ in m and n, and let m', n', and O' be the points conjugate to m, n, and O, and therefore on the curves conjugate to $\mathfrak{A}$ and $\mathfrak{B}$. The points, m', n', and P will evidently be in a straight line, which will be indefinitely near to the line $O'\,P$. The angle made by the lines $m\,Q$ and $n\,Q$, and hence the angle made by $m'\,Q$ and $n'\,Q$ will be infinitesimals of the second order. Therefore, if the directions of $m'\,P$ and $m'\,Q$ make a finite angle, the segment $m'\,n'$ is an infinitesimal of the second order, and the curves conjugate to $\mathfrak{A}$ and $\mathfrak{B}$ are tangent to each other at O'.

13. When the point O' is on an axis, as $P\,Q$, or is at infinity, this demonstration fails. In the first case, the point O is the base point R. It is easy to see that if two curves intersect in an axis, their conjugate curves touch each other in the opposite base point. This is only an apparent exception to the proposition of Art. 11th, for

the two curves tangent at the base point may be considered as intersecting each other in the base point and in a second point indefinitely near to the base point.

14. The curve conjugate to a straight line, $\mathfrak{L}$, passing through one, and only one, of the base points, as P, is a straight line passing through the same base point, P.

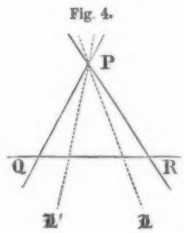


Fig. 4.

For let $\mathfrak{L}$ (Fig. 4) be drawn so that $\mathfrak{L}$ and $\mathfrak{L}'$ shall divide harmonically the angle made at P by the opposite sides of the quadrilateral. Then if O moves along $\mathfrak{L}$, the point O' must move along $\mathfrak{L}'$; that is, if $\mathfrak{L}$ and $\mathfrak{L}'$ are conjugate to each other.

15. If the line $\mathfrak{L}$ is an axis, there is no conjugate line.

For since $P\,Q$ and $P\,R$ divide harmonically the angle made at P by the opposite sides of the quadrilateral, the point conjugate to any point on $P\,Q$ is on $P\,R$. For a like reason the point conjugate to any point on $P\,Q$ is on $Q\,R$. Hence the point of intersection, R, is to be considered the conjugate of every point in $P\,Q$.

It was stated above (Art. 6), that *in general* each point, O, of the plane, $P\,Q\,R$, had one, and only one, conjugate point. The base points, P, Q, and R, are the only exceptions.

16. The curve conjugate to a straight line, not passing through a base point, is a conic that passes through the three base points.

For let a, b, c, d, &c., be points on the straight line, and a', b', c', d', &c., be the points respectively conjugate to them, and let lines be drawn from P and Q to the several points. Then the couples of lines $P\,a$, $P\,a'$; $P\,b$, $P\,b'$; $P\,c$, $P\,c'$, &c., are in involution (*Géom. Sup.*, Art. 244), and hence the anharmonic function of the lines $P\,a'$, $P\,b'$, $P\,c'$, and $P\,d'$ is equal to that of $P\,a$, $P\,b$, $P\,c$, and $P\,d$ (*Géom. Sup.*, Arts. 236 and 13); which is equal to that of $Q\,a$, $Q\,b$, $Q\,c$, and $Q\,d$; which is equal to that of $Q\,a'$, $Q\,b'$, $Q\,c'$, and $Q\,d'$. Therefore the locus of the points a', b', c', d', &c., is a conic passing through P and Q (*Géom. Sup.*, Art. 542), and hence by symmetry through R.

17. Conversely, the conjugate to a conic passing through the three base points, is a straight line that does not pass through a base point.

18. Definitions. — A curve of the nth degree is one that is cut by every straight line in its plane in n points (real and imaginary), and in n points only.

A *conic* is a curve of the second degree.

A *cubic* is a curve of the third degree.

Salmon proposed that the term cubic, used in Algebra to denote an equation of the third degree, should be used in Geometry to denote a curve of the third degree. As we shall have occasion frequently to speak of curves of the fourth degree, we shall make a similar use of the term biquadratic, by which we shall mean a curve of the fourth degree.

19. The curve conjugate to a conic $\mathfrak{C}$ that passes through two, and only two, of the base points, as P and Q, is a conic that passes through the same two base points, and does not pass through the third base point.

It is a conic. For any conic through P, Q, and R cuts the conic $\mathfrak{C}$ in four points; that is, in two points other than the base points, P and Q. Therefore any straight line (Art. 17) cuts the conjugate to $\mathfrak{C}$ in two, and only two, points (real or imaginary), namely, the two points conjugate to the intersections of the first two conics.

Moreover, the conic $\mathfrak{C}$ crosses the axis $P\,R$ in one point, D, other than P. The points conjugate to points indefinitely near to D will evidently be on the line through Q, conjugate to $D\,Q$, and these points will be indefinitely near to Q; in other words, the conjugate to $\mathfrak{C}$ passes through Q, and has for its tangent the line conjugate to $D\,Q$. For a like reason it passes through P. It does not, however, pass through R; for its conjugate, that is, $\mathfrak{C}$, would then be a straight line (Art. 17).

20. The curve conjugate to a conic $\mathfrak{C}$ through one, and only one, base point, P, is a cubic that passes through the other two base points, and has a double point at P.

It is a cubic. For any conic through P, Q, and R cuts $\mathfrak{C}$ in three, and only three, points (real and imaginary) other than P. Hence any straight line cuts the conjugate to $\mathfrak{C}$ in three, and only three, points.

Again, since $\mathfrak{C}$ cuts PQ and PR each in one point other than P, the conjugate to $\mathfrak{C}$ passes through the points R and Q. And since $\mathfrak{C}$ crosses QR in two points, neither of which is a base point, there are two directions in which the conjugate to $\mathfrak{C}$ passes through P; that is, there are two tangents (real or imaginary) to the curve at P, and hence P is a double point. If the tangents are imaginary, then P is an isolated point.

21. The curves conjugate to a conic, $\mathfrak{C}$, that does not pass through any base point, is a biquadratic, having double points at each base point.

It is a biquadratic. For any conic through P, Q, and R cuts $\mathfrak{C}$ in four points (real or imaginary), neither of which is a base point. Hence any straight line cuts the conjugate to $\mathfrak{C}$ in four points (real or imaginary).

Again, $\mathfrak{C}$ cuts each axis in two points, neither of which is a base point. Hence the conjugate to $\mathfrak{C}$ has two tangents (real or imaginary) at each base point.

22. These are single examples of the following general theorem. Let $\mathfrak{A}$ be a curve of the nth order, having a multiple point of the p order at P, a multiple point of the q order at Q, and a multiple point of the r order at R, and let $\mathfrak{A}'$ be conjugate to $\mathfrak{A}$, then,

1st, $\mathfrak{A}'$ is a curve of the $(2n-p-q-r)$th degree; and

2d, It will have at P a multiple point of the $(n-q-r)$th order, at Q a multiple point of the $(n-p-r)$th order, and at R a multiple point of the $(n-p-q)$th order.

1st. $\mathfrak{A}'$ is a curve of the $(2n-p-q-r)$th order.

For any conic cuts $\mathfrak{A}$ in $2n$ points. If the conic passes through P, Q, and R, it will cut $\mathfrak{A}$ in those points respectively, p, q, and r times; and therefore in $(2n-p-q-r)$ other points. Hence, any straight line will cut $\mathfrak{A}'$ in $(2n-p-q-r)$ points.

2d. Since $\mathfrak{A}$ crosses QR in $(n-q-r)$ points other than Q and R, $\mathfrak{A}'$ has $(n-q-r)$ branches passing through P. For a like reason $\mathfrak{A}'$ has $(n-p-r)$ branches at Q, and $(n-p-q)$ branches at R.

These branches may, of course, be imaginary by couples, and when $\mathfrak{A}$ does not pass through P we must, in the above theorem, make $p=o$, and so of the rest.

23. To illustrate the above proposition, let $\mathfrak{A}$ be a cubic.

(a) If $\mathfrak{A}$ passes through neither base point, $\mathfrak{A}'$ is a curve of the sixth degree, having triple points at P, Q, and R.

(b) If $\mathfrak{A}$ passes through only one base point, P, $\mathfrak{A}'$ is a curve of the fifth degree, having a triple point at P, and double points at Q and R.

(c) If $\mathfrak{A}$ passes through P and Q, and not through R, $\mathfrak{A}'$ is a biquadratic having double points at P and Q, and a single point at R.

(d) If $\mathfrak{A}$ passes through P, Q, and R, $\mathfrak{A}'$ will be a cubic passing through P, Q, and R.

(e) If $\mathfrak{A}$ has a double point at P, and does not pass through Q nor R, $\mathfrak{A}'$ is a biquadratic having a triple point at P, and single points at Q and R.

(f) If $\mathfrak{A}$ has a double point at P and a single point at Q, $\mathfrak{A}'$ is a cubic having a double point at P, and single point at Q.

(g) If $\mathfrak{A}$ has a double point at P, and passes through Q and R, $\mathfrak{A}'$ is a conic passing through P, but not through Q nor R.

24. If it be required to construct by points a curve that shall satisfy certain conditions, it may be frequently accomplished by con-

structing points of a conjugate curve that shall satisfy certain corresponding conditions, and then finding the points conjugate to those constructed. This method of construction is especially applicable when the curve is to pass through certain points, or touch certain curves.

25. Thus suppose it was required to construct other points of the conic that passes through five given points.

Take three of the given points for base points, and find the points conjugate to the remaining two points. Then draw through these a straight line, and construct the conjugates to any points on this line. They will evidently be on the conic passing through the five given points (Art. 16).

26. Again, to construct by points a cubic having given one double and six single points of the curve

Take the given double point and two of the single points as base points, and construct the points conjugate to the remaining four points. Through the double point and these four conjugate points construct by points (Art. 25) a conic. The conjugate to the points of this conic will be points of the required cubic (Art. 20).

27. To construct a biquadratic having given one triple point and eight single points.

Take the triple point and two single points as base points, and construct the points conjugate to the remaining six points. Through these construct (Art. 26) a cubic having a double point at the given triple point, and then construct points conjugate to points of this cubic. They will be on the required curve (Art. 23 *e*).

28. To construct a curve of the 5th degree having given one quadruple and 10 single points.

With the quadruple point and two single points as base points, construct the conjugate to the remaining eight points. Through these construct (Art. 27) a biquadratic having the given quadruple

point for a triple point, and construct points conjugate to points of this biquadratic. They will be on the required curve.

29. In like manner may be constructed the curve of the sixth degree having given a multiple point of the 5th order and twelve single points.

And in general by repeating the same process may be constructed a curve of the nth degree having a given multiple point of the $(n-1)$st order, and $2n$ single points.

Moreover, since the conjugate of any point may be constructed by the straight-edge alone (Art. 2), all the preceding constructions may be performed without the aid of the compasses.

30. To construct the curve of the $2n$th degree having given three points of the nth order, one point of the $(n-1)$st order, and $2n$ single points.

Take the three points of the nth order as base points, and the construction depends upon that of Art. 29.

31. In like manner may we construct the curve of the $(2n+1)$st degree having given one point of the $(n+1)$st order, three points of the $(n-1)$st order, and $(2n+1)$ single points.

32. The construction of a cubic that shall pass through nine points is given by CHASLES (*Comptes Rendus*, Tom. XXXVI., Séance du 30 Mai, 1853). Hence, to construct the curve of the nth degree having given a point of the $(n-2)$d order, $(n-3)$ double points and eight single points; take the point of the $(n-2)$d order, a double point, and a single point, as base points, and the construction is made to depend upon that of the curve of the $(n-1)$st degree having a point of the $(n-3)$d order, $(n-4)$ double points and eight single points; that is, on a similar problem to the one required, and of the next inferior degree. Repeating the process, the construction is made to depend at last on the construction of the cubic through nine given points.

[To be continued.]

SECOND PRIZE ESSAY. – ON CENTRAL AND DISTURBING FORCES.*

By DAVID TROWBRIDGE, Perry City, N. Y.

25. ASSUME the *elements* of the planetary orbits to be known: $\pi =$ the longitude of the perihelion, $\Omega =$ the longitude of the ascending node, $i =$ the inclination. Put $\varphi =$ the true, and $E =$ the eccentric, and $M =$ the mean anomaly; $\lambda =$ the true longitude, and $l =$ the true latitude. All these quantities are regarded as heliocentric.

The equations of an ellipse, in terms of the eccentric anomaly, are,

$$x = a(\cos E - e), \qquad y = b \sin E. \tag{71}$$

Whence we obtain

$$\sqrt{(x^2 + y^2)} = r = a(1 - e \cos E). \tag{72}$$

We also have

$$r = \frac{a(1-e^2)}{1 + e \cos \varphi}. \tag{73}$$

Equating these two values of r, and reducing, we find

$$\tan \tfrac{1}{2}\varphi = \sqrt{\frac{1+e}{1-e}} \cdot \tan \tfrac{1}{2} E. \tag{74}$$

The differential coefficients of (71), with respect to the time, are,

$$D_t x = -a \sin E \, D_t E, \qquad D_t y = b \cos E \, D_t E. \tag{75}$$

If we change the axis from which the area is reckoned (Art. 5), equation (12) gives

$$c = x D_t y - y D_t x. \tag{76}$$

Substituting in (76) from (71) and (75), we find

* This Essay is a continuation of the one in Vol. II. p. 150, of the Mathematical Monthly.

$$(77) \qquad c = \frac{2\pi ab}{T} = ab(1 - e\cos E)\, D_t E.$$

If we put $\frac{2\pi}{T} = n$, reduce, and integrate, we have

$$(78) \qquad nt = M = E - e\sin E,$$

since n is the angle described in the unit of time.

To find E in terms of M, we have

$$(78) \quad E = M + e\sin E, \quad \text{and} \quad \sin E = \sin[M + e\sin E].$$

The continual substitution of this value of $\sin E$ in (79) gives the series

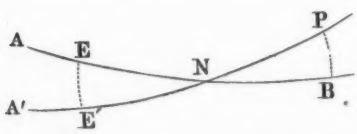
$$(80) \quad E = M + e\sin[{}_{1}M + e\sin[{}_{2}M + e\sin[{}_{3}M + \ldots + e\sin[{}_{\infty}M].$$

If e is small, a few terms only need be employed.

26. From equations (1), (2), (3), (41), and (20) we have

$$(81) \quad D_t^2 x = -\frac{mx}{r^3}, \quad D_t^2 y = -\frac{my}{r^3}, \quad D_t^2 z = -\frac{mz}{r^3}, \quad D_t \varphi = \frac{c}{r^2}.$$

Before we integrate these equations it will be necessary to transform them a little. For this purpose, let ANB be the ecliptic, $A'NP$ the orbit of the planet, N the ascending node, A the Vernal Equinox, E' the perihelion, and P the planet. Then $AN = \Omega$, $AE = \pi$, and $A'N = \Omega'$, $A'E' = \pi'$, the orbit values of the same quantities; $ANA' = i = PNB$, $E'P = \varphi$, $AB = \lambda$, $PB = l$. Put $NP = \beta$, $NB = \beta'$. We then have, by making $E'N = \omega' = \Omega' - \pi'$,



$$(82) \quad \varphi = \beta + E'N = \beta + A'N - A'E' = \beta + \Omega' - \pi' = \beta + \omega'.$$

Differentiating with respect to t, we have

$$(83) \qquad D_t \varphi = D_t \beta;$$

and hence (81) gives $\qquad D_t \beta = \frac{c}{r^2}$.

By Analytical Geometry we have

$$(84) \qquad x = r\cos l\cos\beta', \qquad y = r\cos l\sin\beta', \qquad z = r\sin l.$$

Since PNB is a right spherical triangle, Spherical Trigonometry gives

$$(85) \qquad x = r\cos\beta, \qquad y = r\cos i\sin\beta, \qquad z = r\sin i\sin\beta.$$

(83) and (85) being substituted in (81), give

$$(86) \quad D_t^2 x = -\frac{m}{c}\cos\beta\, D_t\beta, \qquad D_t^2 y = -\frac{m}{c}\cos i\sin\beta\, D_t\beta,$$
$$D_t^2 z = -\frac{m}{c}\sin i\sin\beta\, D_t\beta.$$

Integrating, and we obtain

$$(87) \qquad D_t x = C - \frac{m}{c}\sin\beta, \qquad D_t y = C' + \frac{m}{c}\cos i\cos\beta,$$
$$D_t z = C'' + \frac{m}{c}\sin i\cos\beta.$$

To find the values of the arbitrary constants, we observe that the velocities in the direction of the co-ordinate axes are each $= 0$, when the co-ordinates are each a maximum; hence,

$$D_\beta x = D_\beta r\cos\beta - r\sin\beta = 0, \qquad \text{or} \qquad \frac{D_\beta r}{r} = \tan\beta.$$

But $$\log r = \log a(1-e^2) - \log(1 + e\cos(\beta+\omega'));$$

and hence $$\frac{D_\beta r}{r} = \frac{e\sin(\beta+\omega')}{1+e\cos(\beta+\omega')} = \frac{\sin\beta}{\cos\beta}.$$

Reducing,

$$\sin\beta = e[\sin(\beta+\omega')\cos\beta - \cos(\beta+\omega')\sin\beta] = e\sin\omega'.$$

This value of $\sin\beta$ in the first of (87) gives,

$$0 = C - \frac{m}{c}e\sin\omega', \qquad \text{and} \qquad C = \frac{m}{c}e\sin\omega'.$$

In a similar manner we can find the value of C' and C'', and we shall thus obtain

$$(88)\quad D_t x = \frac{m}{c}(\sin\beta + e\sin\omega'),\quad D_t y = \frac{m}{c}\cos i(\cos\beta + e\cos\omega'),$$

$$D_t z = \frac{m}{c}\sin i(\cos\beta + e\cos\omega').$$

If we put $D_t x = x'$, $D_t y = y'$, $D_t z = z'$, and take the sum of the squares of (88) we find, by recollecting that $\cos(\beta + \omega') = \cos\varphi$,

$$(89)\quad D_t s^2 = v^2 = s'^2 = x'^2 + y'^2 + z'^2 = \frac{m^2}{c^2}[1 + e^2 + 2e\cos\varphi].$$

Put V_p for the velocity at the perihelion, and V_a at the aphelion, then, when $\varphi = 0$ in (89), $V_p^2 = \frac{m^2}{c^2}(1+e)^2$, and

$$(90)\qquad V_p = \frac{m}{c}(1+e).$$

When $\varphi = 180$, $V_a^2 = \frac{m^2}{c^2}(1-e)^2$, and

$$(91)\qquad V_a = \frac{m}{c}(1-e).$$

Whence we obtain

$$(92)\qquad \frac{V_p}{V_a} = \frac{1+e}{1-e} = \frac{\tan^2\frac{1}{2}\varphi}{\tan^2\frac{1}{2}E},$$

by (74).

When the planet is at the mean distance, $\cos\varphi = -e$, and hence

$$(93)\qquad V^2 = \frac{m^2}{c^2}(1-e^2) = \frac{m}{a},$$

since $c^2 = m\,a(1-e^2)$ by (51). Substituting this value of V^2 in (55), making $r = a$, and we have $h = -\frac{m}{a}$. This value of $h = C$ in (42) gives

$$(94)\qquad \frac{m}{a} = \frac{2m}{r} - (x'^2 + y'^2 + z'^2) = \frac{2m}{r} - s'^2.$$

27. The planetary *elements* are dependent on the velocity with which the bodies move, or the space passed over in a given time, and this is dependent on the attractive force of the sun, which is itself a function of the distance. We may hence regard each and

every *element* as a function of the distance, velocity, and time; and if we let A represent any one of the planetary elements, we can have the equations,

$$(95) \qquad A = f(x, y, z, x', y', z', t), \qquad A = f(r, s', t).$$

28. All the preceding 95 equations pertain to the motion of a single planet around the sun. When this is the case, A in (95) is strictly a constant; but if we introduce a second planet, it exercises an influence that causes the value of A to vary; and the extent of this variation will depend on the amount of the disturbing force. If we imagine an ellipse to be described, passing through the position of the disturbed planet (which call P), at any instant, it is evident that by an expansion, or contraction of this ellipse, as the case may require, we can suppose it to successively represent or pass through every point that P will occupy in completing a revolution. In this case it will be seen, that, at any instant (t), the distance of P from the sun, and also its velocity, will be the same in the *variable ellipse*, as in the orbit actually described; but the force that pertains to the two orbits will be different, otherwise the orbit would still be an ellipse. *The difference of the two forces will be the disturbing force.* If A varies, it will vary with the time. If we let D' represent the differential coefficient when the function (95) pertains to an ellipse, and D the differential coefficient when it pertains to the orbit actually described, the differentiation of the first of (95) gives on both suppositions,

$$(96) \quad 0 = (D'_t A) + D'_x A\, D'_t x + D'_y A\, D'_t y + D'_z A\, D'_t z + D'_{x'} A\, D'_t x' + D'_{y'} A\, D'_t y' + D'_{z'} A\, D'_t z'.$$

$$(97) \quad D_t A = (D_t A) + D_x A\, D_t x + D_y A\, D_t y + D_z A\, D_t z + D_{x'} A\, D_t x' + D_{y'} A\, D_t y' + D_{z'} A\, D_t z'.$$

$(D'_t A)$ and $(D_t A)$ are partial differential coefficients.

According to the hypothesis, $(D_t' A) = (D_t A)$, $D_t' x = D_t x$, &c., $D_x' A = D_x A$, &c., $D_{x'} A = D_{x'} A$, &c.; but $D_t' x' = D_t^2 x =$ the force in the ellipse along the axis of x is not the same as $D_t x' = D_t^2 x =$ the force in the real orbit along the axis of x. The difference of (96) and (97) then gives

$$(98) \qquad D_t A = D_{x'} A (D_t^2 x - D_t^2 x) + D_{y'} A (D_t^2 y - D_t^2 y) + D_{z'} A (D_t^2 z - D_t^2 z).$$

If we put

$$(99) \qquad Q X,\ Q Y,\ Q Z,$$

for the disturbing force along the axes, we shall evidently have

$$Q X = D_t^2 x - D_t^2 y,\quad Q Y = D_t^2 y - D_t^2 y,\quad Q Z = D_t^2 z - D_t^2 z.$$

Hence,

$$(100) \qquad D_t A = Q X . D_{x'} A + Q Y . D_{y'} A + Q Z . D_{z'} A.$$

This is a general formula, since it takes into account the whole amount of the disturbing force.

If we assume

$$(101) \qquad A = A_x + A_y + A_z,\ D_t A = D_t A_x + D_t A_y + D_t A_z,$$

such that

$$(102) \qquad D_t A_x = Q X . D_{x'} A,\ D_t A_y = Q Y . D_{y'} A,\ D_t A_z = Q Z . D_{z'} A,$$

we can compute the effects of the disturbing force in the direction of the co-ordinate axes (which we may regard as fixed in space) separately, and then by (101) we can conclude the joint or total effects. This, in general, will simplify the question.

If we treat the second equation of (95) in the same manner as that of the first, we shall find

$$(103) \qquad D_t A = Q S . D_s' A,$$

in which $$Q S = D_t^2 s - D_t^2 s.$$

$Q\,S$ acts along the tangent of the curve, and it is hence the tangential force; and (103) will serve to compute the effects of this force.

29. $Q\,X$, $Q\,Y$ and $Q\,Z$ can represent the sum of the disturbing forces of any number of bodies, or any one of them. If the action of all the planets had to be considered at one time, the problem of perturbation would be very difficult; but from a principle in Mechanics, "immediately flowing from the primary relations between forces and the motions which they produce, that when a number of very minute forces act at once on a system, their joint effect is the sum, or aggregate of their separate effects, at least within such limits that the original relations of the parts of the system shall not have been materially changed by their actions"; and hence it simplifies the question very considerably to consider the action of each disturbing body separately. The question is thus reduced to the famous *Problem of Three Bodies*, which at present can only be solved approximately.

30. In the Solar System, the *inclination* of the planetary orbits to each other is so small, that it introduces little error in supposing the disturbed and the disturbing body to both lie in the same plane, except when we wish to compute the effect on the nodes and inclination.

31. If we substitute $m\,r^{-2}$ for R, and $-m\,a^{-1}$ for $C=h$, in equation (28), and integrate, we find

$$D_t r^2 = \frac{2m}{r} - \frac{m^2}{r^2} - \frac{m}{a} = m\,a\left[\frac{2}{a\,r} - \frac{}{m\,a\,r^2} - \frac{1}{a^2}\right]. \quad \text{But } \frac{c^2}{m\,a} = 1 - e^2.$$

$$\therefore\ D_t r^2 = m\left[\frac{a^2 e^2 - (a-r)^2}{a\,r^2}\right]. \tag{104}$$

If we let $k^2 = a\,(1-e^2)$, and multiply (104) by $\frac{k^2}{e^2}$, we find

$$\frac{k^2 D_t r^2}{e^2} = 1 - \frac{(k^2-r)^2}{e^2\,r^2}. \tag{105}$$

By collecting equations (76) and (94) we have

$$c = x y' - y x', \tag{106}$$

$$\frac{m}{a} = \frac{2m}{r} - (x'^2 + y'^2) = \frac{2m}{r} - s'^2 = \frac{2m}{k^2}[1 + e\cos(\lambda + \pi)] - (x'^2 - y'^2),$$

in which $z' = 0$, in accordance with Art. 30; and $\lambda + \pi = \varphi$ are so chosen that π is the longitude of the perihelion. Since

$$\frac{k^2}{r} = 1 + e\cos(\lambda + \pi), \qquad \cos(\lambda + \pi) = \frac{k^2 - r}{e\,r};$$

and hence

$$\sin(\lambda + \pi) = \sqrt{\left(1 - \frac{(k^2 - r)^2}{e^2 r^2}\right)} = \frac{k\,D_t r}{e}, \tag{107}$$

by (105).

The manner in which we found equation (100) shows that $D_{x'} A$, $D_{y'} A$, $D_{z'} A$, are partial differential coefficients. The first of equations (106) will then give

$$D_{x'} c = -y, \qquad D_{y'} c = x,$$

and these values in (100), regarding $Q Z = 0$, give

$$D_t c = -Q X \,.\, y + Q Y \,.\, x. \tag{108}$$

The second of (106) gives $m D_{x'} a = 2 a^2 x'$, $m D_{y'} a = 2 a^2 y'$. These values in (100) give

$$D_t a = \frac{2 a^2}{m}[Q X \,.\, D_t x + Q Y \,.\, D_t y]. \tag{109}$$

We also have, from (106),

$$D_{x'} e = \frac{k^2 x'}{m\cos(\lambda + \pi)}, \qquad D_{y'} e = \frac{k^2 y'}{m\cos(\lambda + \pi)}.$$

These values, in (100), give

$$D_t e = \frac{k^2}{m\cos(\lambda + \pi)}[Q X \,.\, D_t x + Q Y \,.\, D_t y] = \frac{k^2 e\,r}{2 a^2 (k^2 - r)} \,.\, D_t a. \tag{110}$$

For the variation of π, (106) gives

$$D_{x'}\pi = -\frac{k^2 x'}{m e \sin(\lambda+\pi)} \qquad D_{y'}\pi = -\frac{k^2 y'}{m e \sin(\lambda+\pi)}.$$

These values, substituted in (100), and reducing by means of (107), we find

$$(111)\quad D_t\pi = -\frac{k}{m D_t r}[Q X . D_t x + Q Y . D_t y] = -\frac{k}{2a^2 D_t r} . D_t a.$$

From (59) we have

$$(112)\qquad m T^2 = 4\pi^2 a^3, \qquad D_t T = \frac{6\pi^2 a^2}{m T} . D_t a.$$

The mean angular velocity, or *mean motion*, is

$$n = \frac{2\pi}{T}, \qquad n^2 = \frac{4\pi^2}{T^2}, \qquad n^2 a^3 = \frac{4\pi^2 a^3}{T^2} = m,$$

by (59). Hence,

$$(113)\qquad n^2 = m a^{-3}, \quad \text{and} \quad D_t n = -\frac{3m}{2na^4} . D_t a.$$

If we put M' for the mean longitude at a given epoch, and then $nt + M'$ will give the mean longitude for any time t, after; and, since π is the longitude of the perihelion, $nt + M' - \pi$ will give the mean anomaly at any time; and hence, by (78),

$$(114)\qquad nt + M' - \pi = E - e\sin E.$$

If M' varies, it will be in consequence of the variation of π and e, since nt and E are variable quantities in elliptic motion. Hence,

$$D_t M' = D_t\pi - \sin E\, D_t e = D_t\pi - \sqrt{\left(1 - \left(\frac{a-r)}{a}\right)^2\right)}\, D_t e.$$

By substituting for $D_t\pi$ and $D_t e$, their values as given by (110) and (111), we have

$$(115)\quad D_t M' = -\left[\frac{k}{2a^2 D_t r} + \frac{k^2 e r}{2a^2(k^2 - r)}\sqrt{\left(1 - \left(\frac{a-r}{a}\right)^2\right)}\right] D_t a.$$

32. To investigate the effects of the tangential force, we shall use the equation

$$\frac{m}{a} = \frac{2m}{k^2}\left[1 + e\cos(\lambda + \pi)\right] - S'^2. \tag{116}$$

For a, we have $D_s a = \frac{2a^2 s'}{m}$, and hence, by (103),

$$D_t a = \frac{2a^2}{m} \cdot Q S \cdot D_t s. \tag{117}$$

For e, we have $D_s e = \frac{k^2 s'}{m\cos(\lambda+\pi)} = \frac{k^2 e r s'}{m(k^2 - r)}$. Hence, by (103),

$$D_t e = \frac{k^2 e r}{m(k^2 - r)} \cdot Q S \cdot D_t s. \tag{118}$$

For π, we have $D_s \pi = -\frac{k^2 s'}{m e \sin(\lambda+\pi)} = -\frac{k^2 s'}{m D_t r}$. Hence,

$$D_t \pi = -\frac{k^2}{m D_t r} \cdot Q S \cdot D_t s. \tag{119}$$

$$D_t M' = -\frac{k^2}{m}\left[\frac{1}{D_t r} + \frac{e r}{k^2 - r}\sqrt{\left(1 - \left(\frac{a-r}{a}\right)^2\right)}\right] \cdot Q S \cdot D_t s. \tag{120}$$

The preceding equations expressing the differential coefficients of the elliptic elements with respect to the time, give the momentary variations of these elements, and without the aid of the integral calculus, they are capable of furnishing us accurate ideas respecting the action of the disturbing force.

NOTES AND QUERIES.

I. *Note on a Question in Interest.* — In the March number of the Monthly, Prof. Hoyt cites the following from Robinson's Algebra: — "A man borrowed $ 60; what sum shall he pay daily to cancel the debt, principal and interest, in 60 days; interest at 10 per cent for

12 months of 30 days each?" The answers given in different editions are $\$1\frac{5}{600}$, and $\$1\frac{61}{7200}$; and Prof. HOYT asks, "Is either of them *correct*, or is the true answer $\$1\frac{61}{7259}$?"

Before answering this question, let us consider what rule or principle shall guide us in treating the commercial relations described in the problem. It is required to pay principal and interest. *What* interest? There are in common use two modes of adjusting the interest account, when an obligation is cancelled by partial payments. The one is described by the United States rule; the other by the Vermont Rule. According to the former, the interest to be cancelled is that which shall accrue on the unpaid parts of the principal during the intervals between the successive payments; according to the latter, the interest to be paid is the excess of the interest of the given sum for the whole time, above that accruing on the various payments from the dates at which they were severally made to the time of settlement. These two methods give different results. The problem under consideration involves precisely those commercial relations which require the application of one or the other of these rules. ROBINSON has adopted, in his solutions, neither of these rules, and both of his answers are erroneous. Prof. HOYT has based his solution on the Vermont rule, and has given the correct answer according to that rule. But it seems to me, that in all examples of this nature, where this condition is not defined in the problem, the United States rule should be applied. Not only has it the higher commercial authority, but the analysis by this principle is more pertinent and rational. The given sum is made to draw interest for the first interval. Then a payment is made, which cancels the interest due and a part of the principal. There is now left a balance of the principal on which interest will accrue for another interval; and so on.

To obtain a general solution, let p represent the principal, r the

interest of $1 for one interval, y one of the equal payments, and n the number of payments. Then

(1) $y = \frac{p\,r\,(1+r)^n}{(1+r)^n-1}$, by the United States rule,

(2) $y = \frac{2\,p\,(1+r\,n)}{n\,(n-1)\,r+2n}$ by the Vermont rule.

The answer to this problem by equation (1) is $ 1.0084984+.

It is but just to say that Robinson is not the only author in fault in respect to examples of this kind. In Davies's University Arithmetic we find the following: —

"A farmer bought a piece of land for $ 1,500, and agreed to pay principal and interest in five equal annual instalments; how much was the annual payment?"

A reference to the Key shows that 7 per cent interest is understood. The answer given is $ 360.56. Now this answer cannot be verified by any recognized rule for partial payments. The annual payment required to cancel an obligation of $ 1,500 in five years is $ 366.08, interest reckoned by the United States rule, and $ 342.10, interest reckoned by the Vermont rule. — Prof. J. C. Porter, Liberal Institute, Clinton, New York.

II. *Note on the General Theory of Equations.* — In developing the general theory of equations, it is usual to multiply together the transposed equations formed by x with each of its assigned values, and to treat x in the product, though of different values, as one quantity. The student is often perplexed by this process. Why set down as x^2 the product of x having one value, by x having another value? Why unite the several terms containing the first power of x in the product, when in fact x has different values in those terms? And why will each one of the assigned values of x

verify the resulting equation by being substituted throughout, when the terms which are written as powers of x are the products of factors originally different, or the sums of such products?

In order to explain this, let us at first keep all the monomial factors distinct. Multiplying together $x+a=0$, $x'+b=0$, $x''+c=0$, &c., we obtain the successive equations,

$$(x+a)(x'+b)=0.$$
$$(x+a)(x'+b)(x''+c)=0.$$
$$(x+a)(x'+b)(x''+c)(x'''+d)=0, \text{ \&c.}$$

Now it is evident that whatever be the number of binomial factors used as above, we may substitute x for x', x'', &c., or any one of these for all the others, without destroying the equation; for in each case one of the factors, and, consequently, the whole product, will still remain equal to zero.

For an example, take the second of the above equations. Expanding this, we obtain an equation having eight terms. But since $x+a=0$, we have $(x+a)(x+b)(x+c)=0$; and since $x'+b=0$, $(x'+a)(x'+b)(x'+c)=0$; and since $x''+c=0$, $(x''+a)(x''+b)(x''+c)=0$. Expanding these and uniting like terms, we get from one equation three numerically distinct equations, of four terms each.

$$x^3+Ax^2+Bx+C=0.$$
$$x'^3+Ax'^2+Bx'+C=0.$$
$$x''^3+Ax''^2+Bx''+C=0.$$

All these are obviously contained in the following general formula, where X is to be understood as having three values:—

$$X^3+AX^2+BX+C=0.$$

Thus it appears that the equation obtained by blending x, x', x'',

&c. in the product, though different from that which we should obtain by keeping them distinct, is nevertheless a true equation, and one which each of the originally assigned values of x will verify. — Prof. E. W. Evans, Marietta College, Ohio.

III. *Trigonometrical Formulas.* — The values of $\sin(A \pm B)$, $\cos(A \pm B)$ in terms of the sines and cosines of A and B may be deduced as follows: Take two triangles whose sides are a, b, c, and opposite angles A, B, C, and suppose the angle A in one of them to be obtuse. From the vertical angle C, drop a perpendicular, d, on the base in one case, and on the base produced in the other. The triangles stand on the base AB, and D is the foot of the perpendicular. If $A = CAD$ and $B = CBD$ in both cases, then $\sin C = \sin(A+B)$ in the first, and $\sin C = \sin(A-B)$ in the second, and

$$\sin C = \sin(A \pm B) = \frac{dc}{ab} = \frac{d(DB \pm AD)}{ab} = \frac{d}{b} \cdot \frac{DB}{a} \pm \frac{d}{a} \cdot \frac{AD}{b}$$
$$= \sin A \cos B \pm \sin B \cos A,$$

since, in both cases,

$$\sin A = \frac{d}{b}, \quad \sin B = \frac{d}{a}, \quad \cos A = \frac{AD}{b}, \quad \cos B = \frac{DB}{a}.$$

In the first case, $\cos C = -\cos(A + B)$, and, in the second, $\cos C = \cos(A - B)$. But $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$, and therefore

$$\cos C = \cos(A \pm B) = \mp \frac{a^2 + b^2 - c^2}{2ab} = \frac{AD}{b} \cdot \frac{BD}{a} \mp \frac{d}{b} \cdot \frac{d}{a}$$
$$= \cos A \cos B \mp \sin A \sin B,$$

since

$$b^2 = d^2 + AD^2, \quad a^2 = d^2 + BD^2, \quad c^2 = (BD \pm AD)^2.$$

— Prof. R. M. Moore, M'Kendree College, Lebanon, Illinois.

Mathematical Monthly Notices.

The Claim of LEIBNITZ *to the Invention of the Differential Calculus, by* DR. H. SLOMAN. *Translated from the German with considerable Alterations and new Addenda by the Author.* Cambridge: MACMILLAN & Co., and 23 Henrietta Street, Covent Garden, London. 1860. 4to. pp. 157.

A few years ago PROFESSOR DE MORGAN remarked, that "all the notions as to what is fair and what is unfair in controversy have undergone much change since the commencement of the last century." This remark may be true as to the world in general, but it is evidently false as to DR. SLOMAN. Into the great controversy concerning the invention of the Differential Calculus, which of late years has had so much new interest thrown around it, our author enters with a bitterness that KEILL and FATIO DE DUILLIER might have envied. He espouses the Newtonian side of the contest, and endeavors to turn back the tide which has of late been so steadily setting towards a complete vindication of the fame of LEIBNITZ. The researches of GERHARDT, DE MORGAN, and others, and the new edition of the Commercium Epistolicum by BIOT and LEFORT, were fast establishing the claim of LEIBNITZ to an independent invention of the Differential Calculus, as well as to the sole invention of the notation that gave to it its vast power. But while every true man was rejoicing to see this vindication of a great intellect, our author enters the field with very different feelings. With very little in the way of new facts, and that little unimportant, his book is filled with misrepresentations, garbled extracts, mean insinuations, and base perversions. The only thing it exhibits fairly, it exhibits undesignedly, namely, the moral character of its author, showing it to be of that low type that labors to put the worst construction on every word and act of an opponent.

We do not purpose to enter into an examination of this book in detail. To do so would involve a review of the whole controversy, and would, moreover, ascribe to the work an importance which it does not deserve. We shall, however, examine a preliminary page, which purports to give extracts from GERHARDT, EDLESTON, and LEIBNITZ, and which stands thus: —

GERHARDT: "So it is Leibnitz — — to whom "we owe this. The Chapter in which the question "respecting the first discoverer of the higher An- "alysis has till now been agitated, now disappears "from the history of the mathematical sciences. "The battle of more than one hundred years, "about the first discoverer of the Differential Cal- "culus, is now at its end." *Gerhardt*, II. page 62.

GERHARDT: "So ist es Leibnitz — — dem wir "dies verdanken. Das Capitel, in welchem die "Frage über den ersten Entdecker der höheren "Analysis bisher erörtert wurde, verschwindet for- "tan aus der Geschichte der mathematischen Wis- "senschaften. Der mehr als hundert jährige Streit "über den ersten Entdecker der Differenzialrech- "nung ist zu Ende." *Gerh. Abh.* II. S. 62.

EDLESTON: Synoptical view of Newton's life:

1666 Octob. Small tract on fluxions and fluents with their applications to a variety of problems on tangents, curvature, areas, lengths, and centres of gravity of curves.

1666 November. Small tract similar to the preceding but apparently more comprehensive. (Notation by points in first and second fluxions, Basis of his larger tract of 1671.) *Edleston* in his Correspondence of Sir I. Newton, 1850, page 21.

LEIBNITZ: Cum Parisios apulissem anno Christi 1672 eram ergo — — in pene dixerim superba Matheseos ignorantia. *Leibnitz* in Gerhardt's Pamphlet, I. page 29 et 30, line 2.

The reader would suppose from this, that GERHARDT here claims that LEIBNITZ was "the first discoverer of the Differential Calculus," which claim is disposed of by showing, in the succeeding extracts, that NEWTON had his method of Fluxions in 1666, six years before the time

when LEIBNITZ declared his ignorance of Mathematics. We pass over the mistakes in the last extract, of *apulissem* for *appulissem*, and *ergo* for *ego*, and also the bringing together into one sentence of the parts of widely separated sentences, because it happens that here the sense is not essentially changed. But it is otherwise with the first extract. The first words, "So ist es LEIBNITZ," are not in GERHARDT; the next four, "dem wir dies verdanken," are altered from a sentence on the preceding page, and the remainder of the extract is part of a sentence separated from the rest by more than half a page. The meaning of GERHARDT is, by this ingenious process, entirely perverted. The passage occurs in a chapter headed, "*The Discovery of the Algorithm of the Higher Analysis by* LEIBNITZ," and has reference to the system of notation invented by him. No one could mistake this; for GERHARDT'S words in the first sentence quoted from are, "the *notation* which we owe to Leibnitz" (*die Bezeichnungsweise die wir Leibnitz verdanken*). GERHARDT does not deny, and no one, we presume, denies, that NEWTON had made the discovery of what was afterwards called his Method of Fluxions (but without any general notation) several years before LEIBNITZ began the study of the higher mathematics; and his object in this place is simply to claim for LEIBNITZ the invention of his system of notation. After showing from his manuscripts the successive steps in the development of the subject, as they presented themselves to the mind of LEIBNITZ, GERHARDT stops a moment to consider the importance of the invention, and closes with the opinion, that every suspicion of plagiarism on the part of LEIBNITZ must now vanish, and, therefore, that the long controversy may now be considered at an end. We give below a translation of the whole passage, — interesting in itself, — that our readers may judge for themselves how fairly the extracts of Dr. SLOMAN are made.

"This is the origin of that notation to which the Higher Analysis owes its growth and wonderful perfection, — and the twenty-ninth of October, 1675, is the memorable day that saw its birth. The importance of the subject demands that we should pause an instant, in order to rightly estimate this significant moment. The bold speculations of KEPLER and the *Method of Indivisibles* of CAVALIERI had furnished the means of treating more easily than before the problems of higher geometry. In addition, WALLIS, in his *Arithmetica Infinitorum*, had shown how the general methods of representing magnitudes, introduced by VIETA and DESCARTES, could be used in quadratures and cubatures. By these means a great many particular results were reached that are now obtained by the Higher Analysis; but there was wanting a common bond to free them from their isolation, and unite them under a single point of view. This the natural and most happy notation which we owe to LEIBNITZ effected at a single stroke. It has been universally acknowledged by the great mathematicians of succeeding times, that the Algorithm of LEIBNITZ called forth the surprisingly rapid progress that was made in enlarging and applying the Higher Analysis, and brought about the completion of that truly noble edifice, to survey which and to explore whose various parts are given only to him in whom eminent ability is found united with the highest power of attention and unwearied perseverance. It is not to be overlooked, that LEIBNITZ, led by his lucky genius, came almost accidentally upon the introduction of the algorithm of the Higher Analysis; nor, on the other hand, can it be denied, that being prepared for it by his earlier studies, and surveying the field of science, he recognized at once, though not in its full extent, the importance of the happy discovery, and knew how to employ it in the most skilful manner. This, however, every one must openly concede, — that LEIBNITZ, in introducing his new algorithm, proceeded independently throughout, without even the slightest external influence; an unprejudiced examination of his manuscripts forces us to this conclusion. All doubt, every suspicion of plagiarism, is removed by the manuscripts which are here for the first time connectedly published, and the chapter in which the question respecting the first discoverer of the Higher Analysis has been till now agitated, henceforth disappears from the history of the mathematical sciences. The battle of more than one hundred years about the first discoverer of the Differential Calculus is now at an end."

Let us hope that the spirit which animates the work of DR. SLOMAN will have few imitators. Let us rather cherish and defend the fame of both NEWTON and LEIBNITZ, recollecting that the honor to our race is doubled, if two, instead of one, are shown to be capable of a great invention. Let us be ready to rejoice whenever the impartial hand of later research reveals a new brightness in those great names which the envy and malice of contemporaries sometimes obscure.

H.

Worcester's Quarto Dictionary The Standard

VERY SIGNIFICANT FACTS.

The following recommendations are from some of the most distinguished American and English scholars. They are but a few from many which have been received, testifying to the superiority of Worcester's Quarto Dictionary. These testimonials are of the highest value, for they have all been given during the present year, and after an examination of this work and of that which is endeavoring to hold the position of a rival. The scholars of America and of England, with scarcely an exception, have decided in favor of Worcester. Not a single scholar, equal in authority to any one mentioned below, can be cited as giving, after a comparison of the two works, the preference to Webster's Dictionary. We give the testimony: —

From C. C. FELTON, LL.D., *President of Harvard College.*

Aware of the labor and care which had been devoted to this (the department of scientific terms) as well as to other parts of the work, I felt assured that Worcester's Quarto Dictionary would more nearly meet the public wants than any other hitherto published.

My expectations have been more than fulfilled. I find it not only rich beyond example in its vocabulary, but carefully elaborate in all the details, and thoroughly trustworthy as a guide to the most correct and elegant usage of the language.

From the REV. JOSEPH BOSWORTH, D.D., F.R.S., *Professor of Anglo-Saxon, Oxford, England.*

It is the most complete and practical, the very best, as well as the cheapest English Dictionary that I know.

From GEORGE P. MARSH, LL.D.

The work of Dr. Worcester is unquestionably much superior to any other general dictionary of the language in every one of these particulars (orthography, pronunciation, definition, fulness of vocabulary, and precision and distinctness of definition).

From REV. W. WHEWELL, D.D., *Master of Trinity College, England.*

I have repeatedly consulted the Dictionary since it has been in my possession, and have seen reason to think it more complete and exact than its predecessors.

From CHARLES RICHARDSON, LL.D., *the oldest living English Lexicographer, England.*

I sincerely hope you may enjoy from your brethren, both in America and England, that tribute of honor to which you have earned so undoubted a title.

From D. R. GOODWIN, D.D., *President of Trinity College, Hartford.*

It was but a short time since that I was led to commend another dictionary as, on the whole, and with some exceptions, the best and most complete thing of the kind within my knowledge. The commendation was honestly given at the time; but now *it must be withdrawn in favor of yours.* I consider your dictionary, in orthography, pronunciation, and definitions, as superior to any of its predecessors.

From REV. W. B. SPRAGUE, D.D., *of Albany, N. Y.*

My opinion of Worcester's Quarto Dictionary, after having given it as extended an examination as my circumstances would admit, is, that there is no other dictionary in the language that compares with it for completeness, accuracy, comprehensiveness, and precision, and perhaps I ought to add, that I have arrived at this conclusion rather contrary to a preconceived opinion.

From REV. HENRY A. BOARDMAN, D D., *of Philadelphia.*

I particularly like it (the Dictionary), 1. Because of its very comprehensive character; 2 Because it adheres to the *settled orthography* of our noble language, — discarding those innovations which, however countenanced by certain publishing-houses, have never to any extent been accepted by the scholars of our country.

From LOUIS AGASSIZ, LL.D.

It is of great importance, when the nomenclature of science is gradually creeping into common use, that an English lexicon should embrace as much of it as is consistent with the language we speak. I am truly surprised and highly delighted to find you have succeeded far beyond my expectations in making the proper selection, and combining with it a remarkable degree of accuracy. More could hardly be given except in a scientific cyclopædia.

The following lines are quoted from Harper's Magazine for September. They serve to show very truthfully the comparative value of recent and old commendations: —

"INJUSTICE. — Our attention has been called to a species of injustice of which publishers are sometimes guilty, in publishing commendations of school-books, without giving the dates when they were written. Especially does this merit reproval when these commendations are old, and when it is known that the writers have subsequently commended other and later publications in the same department. It will readily be seen that this is frequently not only an act of injustice to teachers who have had the courtesy to commend a book, but that it is also a fraud upon the public."

SWAN, BREWER, & TILESTON,

131 Washington Street, Boston.

No. 117 Washington Street, Boston, October, 1860.

CROSBY, NICHOLS, LEE & CO.

HAVE JUST PUBLISHED

THE WORKS OF CHARLES LAMB. With a fine Portrait. In Four Volumes. 12mo. Price, $5.00.

This new edition of Lamb is reprinted from the latest London edition, edited by Talfourd, contains his Life, Letters, and Final Memorials, his Essays of Elia, Rosamund Gray, with other Essays and Poems, being the best and most complete edition issued.

It is elegantly printed on paper slightly tinted, and is in every respect an edition worthy of the author.

"This is the most convenient and elegant edition of Lamb's Works which has been published, either in England or in this country, and it leaves nothing to be desired in respect to typographical beauty or any other essential of a good library edition." — *North American Review.*

The Three Great Histories, — Hume, Gibbon, and Macaulay.

Elegant Library Editions, beautifully printed on tinted paper, convenient in size, and withal the best and neatest yet published.

THE HISTORY OF ENGLAND, FROM THE INVASION OF JULIUS CÆSAR TO THE ABDICATION OF JAMES THE SECOND, 1688. By DAVID HUME, Esq. A new Edition; with the Author's last Corrections and Improvements. To which is prefixed a short Account of his Life, written by himself. In 6 volumes. 12mo. With a Portrait. Price, $7.50.

SIR JAMES MACKINTOSH *says of Hume:* "His greatest work, and that which claims most attention, was his 'History of England,' which, notwithstanding great defects, will probably be at last placed at the head of historical compositions. No other narrative seems to unite, in the same degree, the two qualities of being instructive and affecting. No historian approached him in the union of the talent of painting pathetic scenes with that of exhibiting comprehensive views of human affairs. . . . . His narrative is flowing and various; in common events, short and clear; in great actions, rapid and animated; in affecting incidents, circumstantial and picturesque. His general observations seem always to be required by his subject; the most profound ideas are clothed in a transparent simplicity."

THE HISTORY OF THE DECLINE AND FALL OF THE ROMAN EMPIRE. By EDWARD GIBBON, Esq. With Notes by the Rev. H. H. MILMAN. A new edition. To which is added a complete Index of the whole Work. In 6 volumes. 12mo. With a Portrait. Price, $7.50.

Of the various editions of GIBBON this of MILMAN is uniformly acknowledged to have no rival.

PROF. SMYTH, *in his Lectures on Modern History,* says: "But the whole is notwithstanding, such an assemblage of merits, so various, so interesting, and so rare, that the History of the Decline and Fall must always be considered as one of the most extraordinary monuments that have appeared of the literary powers of a single mind, and its fame can perish only with the civilization of the world."

THE HISTORY OF ENGLAND FROM THE ACCESSION OF JAMES II. By THOMAS BABINGTON MACAULAY. In 4 volumes. 12mo. Price, $5.00.

"It is as fluent and as much colored as Livy, as close and coherent as Thucydides; with far more real condensation and a larger thoughtfulness than either; and quite free from the affected laconisms and sarcasms and epigrams of Tacitus. I do not know that I ever read anything so good as the first forty pages; so clear, comprehensive, and concise, so pregnant with deep thought, so suggestive of great views, and grand and memorable distinctions." — LORD JEFFREY.

CURIOSITIES OF LITERATURE. By ISAAC DISRAELI. With a View of the Life and Writings of the Author, by his Son. In 4 volumes. 12mo. Price, $5.00.

"It embraces a more copious selection of noteworthy facts in regard to books and authors than can be found in almost any other compilation of the kind, while it doubtless owes much of its popularity with all classes of readers to the agreeable form in which these facts are presented." — *North American Review, April,* 1860.

THE NORTH AMERICAN REVIEW. No. CLXXXIX. — For October, 1860.

CONTENTS. — Article I. Homer and his Heroines. II. Climatology. III. Life and Labors of Thomas Prince. IV. Edmund Waller. V. Lord Shaftesbury. VI. Second Volume of Palfrey's "History of New England." VII. Quarantine and Hygiene. VIII. Rush's Occasional Productions. IX. The English Language in America. X. The Origin of Species. XI. An "Inglorious Milton." XII. Critical Notices. XIII. New Publications.

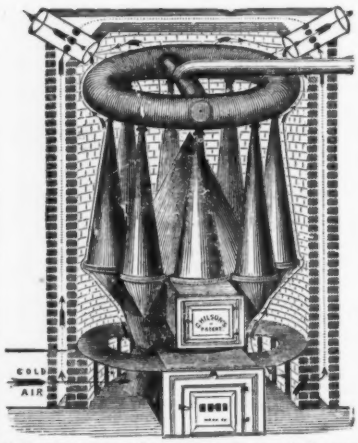
This Review has for nearly fifty years stood at the head of the literature of our country, and has numbered among its contributors most of our eminent literary men. The writings of Webster, Everett, Bancroft, and Prescott were first made known through this work; and its pages are now filled by articles from our most prominent writers.

The NORTH AMERICAN REVIEW is published quarterly, in numbers of nearly three hundred pages each, at five dollars a year.

CHILSON'S
PATENT CONE FURNACE.

PATENTED IN

AMERICA, ENGLAND, AND FRANCE.



THE subscriber begs to call the attention of scientific gentlemen and others to his discovery of a principle for burning fuel, by which all the combustible properties of the smoke and gases eliminated from the burning fuel are thoroughly consumed and their heat retained and made available to the fullest extent, so that there shall be no waste whatever. This important object is accomplished by the use of a large cluster of cones or tapering radiators placed over a broad, shallow, flaring fire-pot, and surmounted by a radiating and exhaustive annular chamber, as represented in the cut. The cones being large at their base and small at the tops, causes the smoke and gases to be suspended directly over and near the fire, where they are thoroughly consumed, and their heat forced to impinge against and up the cones, thereby creating a powerful radiation.

This plan of furnace secures a fresh, healthful heat, free from that scorching, dry, disagreeable, red-hot iron heat and coal-gas so common to hot-air furnaces generally. Persons interested in these matters will here find an invention worthy of their attention, which saves from one half to two thirds of the fuel usually consumed.

COOKING RANGES.

CHILSON'S PATENT DOUBLE OVEN RANGE.

The most important Improvements yet made in Ranges.

THE highly satisfactory testimony received from the large number of these Ranges in use in the past three years should certainly induce those in want to examine this Range, which is *entirely new in principle*, and different from any other before known. It has been got up very perfect and substantial, so that it commends itself at once to the purchaser, in preference to anything else of the Range kind in this country.

Its great economy in the use of fuel, its quick and even baking ovens, its nice arrangement of flues, — without the usual vexatious complication of dampers, — its cast-iron flue plates, preventing any loss of heat in the brick-work, its nice arrangement for roasting ments in the ventilated ovens, &c., will be appreciated by every housekeeper. Sizes in variety for dwellings, hotels, &c., with or without Water Backs for heating water for bathing, &c., Hot Closets, and Hot Air Fixtures for heating additional rooms.

Also, a full assortment of

Chilson's Patent Trio Portable Furnaces, Trio Stoves, Parlor Stoves, Registers, Ventilators, &c., &c.

WAREROOMS 99 & 101 BLACKSTONE ST., BOSTON.
FOUNDERY AT MANSFIELD, MASS.

GARDNER CHILSON.

Philosophical Instruments
AND
SCHOOL APPARATUS,

MANUFACTURED BY

E. S. RITCHIE, BOSTON, MASS.

Few branches of study in our schools are of more practical importance than Natural Philosophy, and it is only by the use of proper apparatus that its principles can be adeqnately elucidated.

Until a comparatively recent period, the attention bestowed upon this subject has been confined to a few of the elementary principles; of late years, however, the growing interest in physical science, as a branch of common education, has created a desire for more ample and varied means of experimental demonstration.

It has been my earnest desire and effort to keep pace with the progress of natural science, and, as new principles have been discovered or developed, to devise and construct new instruments, to improve the old, and to add to my list many which heretofore could only be procured from Europe; and also that, in superior quality and finish, as well as in graceful form and proportions, whatever goes from my manufactory shall take rank with the best products of European make.

So numerous and important are the changes that have consequently been made, that I have published a new and enlarged **Catalogue of 100 pages octavo,** containing descriptions of over

Twelve Hundred Pieces of Apparatus,

and illustrated by over **Three Hundred** finely executed engravings, and also many commendatory letters from the most distinguished Physicists in the country.

E. S. RITCHIE.

Philosophical Rooms, No. 313 Washington Street, Boston.

☞ Catalogues will be sent, prepaid, by mail to order, — please enclose 25 cents in stamps.

PHOTOGRAPHING ON WOOD.

BRICHER AND RUSSELL,

Photographers, Designers, and Engravers

ON WOOD.

129 WASHINGTON STREET, BOSTON.

BRICHER AND RUSSELL, by their new process of Photographing on Wood, are now enabled to produce much more accurate representations of objects than by the old and slow process of hand drawing. They will furnish Portraits, Views of Colleges, Seminaries, Academies, and Public Buildings generally, College and Society Seals, Philosophical Apparatus, Mathematical Instruments, and Diagrams (see specimens in Mathematical Monthly), Illustrations for School Books of all kinds, &c., &c.

Directions.

Get a Photographer to take a *negative* of the object you wish to have engraved, and of the required size, — that is, send us a photograph of the required size, and the *reverse* of what is wanted when printed.

If a negative photograph cannot be obtained, get as sharp and clear an ambrotype as possible.

GRAY'S BOTANIES,

For Schools and Colleges, etc.

By PROF. ASA GRAY, M.D.,

OF HARVARD UNIVERSITY, CAMBRIDGE.

I.	How Plants Grow, 500 Engravings,	Retail, $ 0.75	V.	Manual, Illustrated,	Retail, $ 2.50
II.	Lessons in Botany, 360 "	" 1.00	VI.	Structural and Systematic Botany, 1800 Illustrations,	" 2.00
III.	Manual of Botany,	" 1.50			
IV.	Manual and Lessons in one volume,	" 2.00			

The price of the Manual and Lessons in 1 vol., for schools, academies, &c., is just reduced to $ 2 retail.

☞ Single copies of those used in classes sent Teachers prepaid for *half price*, for examination with a view to introduction. To other persons, they are sent prepaid on receipt of price.

First Supplies for classes sold or exchanged for unsatisfactory books on liberal terms.

From FIFTEEN OCTAVO PAGES OF TESTIMONIALS and reviews from the highest sources in Natural Science, we present several of late date: —

Prof. AGASSIZ: "I have just received a copy of 'Gray's Lessons,' and read it carefully. I am so much pleased with the plan of the work and its execution, that I truly hope it may be generally adopted as a text-book wherever Botany is taught."

Prof. PEARSON, of Union College, writes of the 'Manual': "It is beyond comparison *the best Manual* in this country, and should find a place in every institution where Botany is studied."

Dr. WILLIAM TULLY, formerly Professor of Materia Medica in Yale College, writes: "Having had varied experience as a teacher of Botany for about thirty years, to pupils of both sexes, *I am of the opinion that the botanical works of Professor Gray ought to supersede all others*, not only in our colleges, but also in all the schools of the country."

Rev. HENRY WARD BEECHER, Brooklyn, N. Y.: "I have no hesitation in saying that among all authors who have written on this subject, I esteem Prof. Gray, beyond all comparison, first."

Many testimonials are crowded out, from the following and other distinguished Naturalists and Educators, viz.: —

Dr. LINDLEY, of London, who stands among the first of living Botanists.
Dr. HOOKER, the celebrated Botanist of London.
Pres. HITCHCOCK, of Amherst College.
Prof. TUCKERMAN, Amherst College.
Prof. HOLTON, Middlebury College.
Prof. CHADBOURNE, Williams College.
Prof. JOHN TORREY, of United States Assay Office.
Prof. SCHMIDT, Columbia College.
Prof. A. GUYOT, Princeton College.
Dr. WM. DARLINGTON, of Philadelphia.
Prof. SILLIMAN, Yale College.
Prof. HENRY, of the Smithsonian Institute.
Pres. ANDREWS, Marietta College, Ohio.
Pres. NOTT, Union College.
Dr. WILLIAM TULLY, late Professor of Materia Medica in Yale College.
Prof. WINCHELL, University, Michigan.
Prof. WM. HINCKS, F.L.S., Professor Natural History, Univer. College, Toronto, C. W.
Prof. DANA, Yale College.
Rev. Dr. CURTIS, of North Carolina.
W. J. MARTIN, Prof. Natural Science University of North Carolina.
Dr. CHAPMAN, Author of the "Southern Flora."
Prof. J. S. SCHANCK, D.D., Princeton College.

With many other Presidents and Professors of Colleges, State Superintendents of Public Instruction, State Geologists, Principals of Normal Schools, and hundreds of Principals of the *best Academies, Seminaries, High Schools*, etc.

From the *Atlantic Monthly* (August, 1858): "No man can deny that Dr. Gray's books are all of the highest order of merit. . . . . In the 'Manual,' this clearness of style extends to the judicious selection of distinctive marks, whereby allied species may be distinguished from each other. . . . . Gray's 'Structural and Systematic Botany' has become a standard wherever our language is read.

"For those who do not wish to pursue the study so far, the 'First Lessons' is one of the most happily written scientific text-books ever published.

"'How Plants Grow' is a remarkably successful effort of a truly learned man to write a book actually adapted to young children."

The *North American Review*, in an extended review of the series, says: "Standing as Dr. Gray does at the head of the science in our own country, and scarcely inferior to any botanist of the Old World, we consider it a subject of congratulation that he has found time to write these books especially for the young. The 'How Plants Grow' may be truly called a wonderful book. When we consider that this quarto of two hundred and thirty-three pages, profusely and beautifully illustrated, is offered at the extremely low price of seventy-five cents, we do not hesitate to assert that it is the cheapest botanical work ever issued from the press of any country."

The *American Journal of Education*, edited by HENRY BARNARD, LL.D., Chancellor of the University of Wisconsin, says: "We regard Dr. Gray's works as standing above the ordinary position and rivalry of school-books. It is one of the few instances where a scientific man of the very highest rank has devoted himself to the preparation of a text-book."

From the *National Quarterly Review* (N. Y.), June, 1860: "Dr. Gray's series of Botanies have elicited the unqualified praise of the most eminent botanists of Europe and America. Most of them have been adopted as text-books in several of the principal seminaries and academies of the United Kingdom. At home we believe they are almost universally used, both in our schools and colleges. The features in which they excel all similar works, are their fulness, variety, excellence of arrangement, and accuracy and lucidity of definition."

The (London) *Gardener's Chronicle*, edited by the celebrated Dr. LINDLEY, says: "Never before was a guide to the study of plants written with such admirable simplicity, combined with scientific exactness."

Want of space excludes high commendations from several State Educational journals, the Dublin Journal of Natural History (the highest authority in this department in Great Britain), Harper's Magazine, Silliman's Journal of Science, the American Horticulturist, the London Athenæum, the Medical Eclectic, the American Medical Times, North American Medico-Chirurgical Review, the Methodist Quarterly Review, New England Farmer, N. Y. Observer, Evangelist, Independent, Examiner, Christian Advocate and Journal, Churchman, and other leading religious and literary journals.

IVISON, PHINNEY, AND COMPANY,

Educational Publishers, New York.

ROSS' AMERICAN SCHOOL FURNITURE WORKS,

Established, 1838.

The Pioneer and most Extensive Establishment in the United States.

The above Engraving represents Ross' Improved High School, Normal School, or Academy Double-Column Desk and Chairs; Falls to lift; plain top.

The above Engraving represents Ross' Improved High School, Normal School, or Academy Single-Column Desk and Chair; Fall to lift; plain top.

The above Engraving represents Ross' Improved High School, Normal School, or Academy Single Desk and Chair; Fall to lift; plain top.

The above Engraving represents Ross' Improved High School, Normal School, or Academy Double Desk and Chairs; Falls to lift; plain top.

An Illustrated Catalogue and Information forwarded, on Application, by Mail or Otherwise.

N. B. *Every Article of School Furniture from this Establishment* WILL BE WARRANTED. *All Communications may be addressed to*

JOSEPH L. ROSS,
Office--CHARDON, opp. HAWKINS STREET,
(*Near the Revere House,*) **BOSTON, Mass.**

Entered according to Act of Congress, in the Year 1860, by Joseph L. Ross, in the Clerk's Office of the District Court of Massachusetts.

MATHEMATICAL BOOKS,

IMPORTED AND FOR SALE BY

SEVER AND FRANCIS, CAMBRIDGE,

SUCCESSORS TO JOHN BARTLETT.

ENGLISH.

American Ephemeris and Nautical Almanac. For the Years 1855–62. Each $1.50

BAKER. Treatise on Statics and Dynamics 0.30

BLAND. Algebraical Problems. 8vo. Second-hand 1.50

——— Key to do. 8vo. do. 1.75

——— Philosophical Problems. 8vo. do. 1.50

——— Geometrical Problems. 8vo. do. 1.50

BOOLE. Treatise on Differential Equations 4.00

BOOTH. Theory of Elliptic Integrals. 8vo. 2.25

BORDEN. System of Railway Formulæ. 8vo. 2.25

BRITISH NAUTICAL ALMANAC. 1861 and 1862, each . . . 1.00

CARMICHAEL. Treatise on the Calculus of Operations. 8vo. . 2.75

CHAUVENET. Trigonometry. 8vo. 1.50

COURTENAY. Calculus. 8vo. 2.50

COOMBE. Solutions of the Cambridge Problems, 1840, 1841. 8vo. . 2.25

DEMORGAN. Differential and Integral Calculus 2.00

EARNSHAW. Dynamics. 8vo. 4.25

FISCHER. Logarithmic Tables of Seven Places. Translated from Bremiker's Vega. 8vo. Hf. mor. 2.50

FROST. Mathematical Questions of the Senate-House Examination Papers. 1838–1849. 8vo. 3.00

GAUSS. Theoria Motus. Translated by C. H. Davis. 4to. . . . 5.00

GREGORY. Differential and Integral Calculus by Walton. 8vo. . 5.50

HADDON. Differential Calculus 30

HAMILTON. Lectures on Quaternions. 8vo. 6.30

HANN. Integral Calculus 30

HEATHER. Descriptive Geometry 30

HEMMING. Differential and Integral Calculus. 8vo. 2.50

HUTTON. Mathematics 4.00

——— Recreations. 1 vol. 8vo. 4.00

——— Mathematical Tables. 8vo. 3.75

HYMER. Plane and Spherical Trigonometry 2.50

——— Algebraic Equations 3.00

——— Treatise on Conic Sections. 8vo. 2.75

——— Differential Equations and Calculus of Finite Differences. 8vo. 3.60

——— Integral Calculus 3.00

——— Geometry of Three Dimensions. 8vo. 3.00

JAMIESON. Solutions of the Senate-House Rider. 8vo. 2.25

LARDNER. System of Algebraic Geometry. 8vo. Second-hand . 1.75

MAIN and BROWN. Indicator and Dynometer 1.40

Mathematical Monthly. Edited by J. D. Runkle. Vol. I. In Nos. 3.00

Do. do. Hf. mor. 4.00

Mathematical Problems and Examples of the Senate-House Examination Papers. 1821–1836. With an Appendix containing the Senate-House Questions for 1837. 8vo. 3.00

PARKINSON. Elementary Mechanics. Post 8vo. 2.87

PEIRCE. Analytic Mechanics. 4to. 7.50

PEIRCE, J. M. Analytic Geometry. 8vo. 1.50

PHEAR. Elementary Hydrostatics. Post 8vo. 1.62

PRICE. Differential and Integral Calculus. 3 vols. 8vo. . . . 14.50

Quarterly Journal of Pure and Applied Mathematics. Sylvester, Ferriers, & Co. $6.00 per annum, *post-paid*.

RANKINE. Manual of Applied Mechanics 2.00

SALMON. Conic Sections 3.50

——— Higher Plane Curves 3.50

——— Higher Algebra 1.80

Solutions of the Cambridge Problems. 1848–1851. Ferrers and Jackson. 8vo. 4.50

——— 1854. Walton and McKenzie. 8vo. 2.50

——— 1857. Campion and Walton. 8vo. 2.50

——— Senate-House Riders. Jamieson. 8vo. 3.25

SPOTTISWOODE. Elementary Theorems relating to Determinants. 4to. 1.50

TATE. Exercises on Mechanics, with Key. 2 vols. 1.62

——— Mechanical Philosophy. 8vo. 3.25

——— Materials. 8vo. 1.75

TATE and STEEL. Dynamics. 8vo. 3.00

TODHUNTER. Analytical Statics. Post 8vo. 3.00

——— Differential and Integral Calculus. 2 vols. Post 8vo. 6.00

WALTON. Problems illustrative of Plane Coördinate Geometry. 8vo. 4.80

——— Mechanical Problems. 8vo. 5.50

——— Problems in Hydrostatics and Hydrodynamics. 8vo. . . 3.00

——— On the Differential Calculus. 8vo. 3.25

WALTON and MCKENZIE. Solutions of the Cambridge Problems. 1854. 8vo. $3.25. Do. 1857. . . . . 2.50

——— Problems in Elementary Mechanics. 8vo. 3.00

WAUD. Algebraic Geometry. 8vo. Hf. cf. (Second-hand.) . . 1.50

WHEWELL. Analytic Statics. 8vo. 2.25

WILSON. Dynamics. 8vo. 2.87

WOOLHOUSE. Differential Calculus 30

FRENCH, &c.

AUDÉ. Poussée des Terres. 8vo. Hf. cf. $1.87

BELANGER. Résumé de Leçons de Géométrie Analytique et de Calcul Infinitésimal. 8vo. 2.75

BIOT. Traité des Equations differentielles. Hf. mor. 2.00

BOURCHARLAT. Eléments de Calcul Differential et du Calcul Integral. 8vo. 3.00

BONNET. Leçons de Mécanique Élémentaire à l'Usage des Candidates : l'École Polytechnique. 8vo. 2.00

BRIOSCHE. Théorie des Determinants. 8vo. 2.00

BOURDON. Application de l'Algébre à la Géométrie. 8vo. Hf. cf. 3.00

CALLET. Tables de Logarithms. 8vo. Hf. cf. 5.00

CHASLES. Traité de Géométrie Supérieure. 8vo. Hf. cf. . . 6.00

CHOQUET. Traité élémentaire d'Algébre. 8vo. Hf. cf. . . . 2.00

COURNOT. Traité élémentaire de la Theorie des Fonctions, et du Calcul Infinitésimal. 2 vols. 8vo. 4.00

CRELLE. Journal fur die reine und augewandte Mathematik. Per annum 4.00

DELISLE. Géométrie Analytique. 8vo. Hf. cf. 3.00

DUHAMEL. Cours d'Analyse Calcul Infinitésimal. 2 vols. 8vo. Hf. mor. 5.00

——— Cours de Mécanique. 2 vols. 8vo. Hf. mor. . . . 4.50

DUPIN. Application de Géométrie et de Mécanique à la Marine aux Ponts et Chaussées. 4to. Hf. mor. 4.50

——— Developpments de Géométrie. 4to. Hf. mor. 4.50

——— Géométrie et Mécanique, appliquée aux Arts. 8vo. Hf. mor. 2.50

FRANCŒUR. Cours complet de Mathématiques pures. 2 vols. 8vo. 5.00

——— Éléments de Statique. 8vo. Hf. cf. 2.00

GAUSS. Méthode des Moindres Carrés. 8vo. Hf. cf. 1.62

KÖHLER. Logarithmisch Trigonometrisches Handbuch. 8vo. . 2.12

LACROIX. Traité du Calcul Differential et du Calcul Integral. 3 vols. in 4to, avec 18 pl. Mor. 35.00

LAGRANGE. Mécanique Analytique. 2 vols. 4to. 13.00

——— Théorie des Fonctions Analytiques. 4to. Hf. mor. . 6.00

——— Traité des Equations Numeriques de tous les Degrés. 4to. Hf. mor. 5.00

LALANDE. Tables de Logarithmes. 18mo. 1.25

LAPLACE. Système du Monde. 4to. Hf. mor. 5.50

LEGENDRE. Traité des Fonctions Elliptiques. 3 vols. 4to. Hf. mor. 20.00

LEROY. Traité de Stéréotomie comprenant les Applications de la Géométrie Descriptive à la Theorie des Ombres, la Perspective Lineaire, la Gnomonique, la Coupe des Pierres, et la Charpente. In 4to, avec Atlas de 74 pl. in fol. Paper, $9.00 ; hf. mor. 12.00

——— Traité de Géométrie Descriptive. In 4to, avec Atlas de 71 pl. Paper, $3.00 ; hf. mor. $5.00 and 6.00

——— Analyse Appliquée à la Géométrie des trois Dimensions. 8vo. Hf. cf. 2.25

LIOUVILLE. Journal de Mathématiques Pures et Appliquées. 1836–1857. 22 vols. 3.00

MAHISTRE. Cours de Mécanique Appliquée. 8vo. 3.00

MONGE. Application de l'Analyse à la Géométrie. 4to. Hf. mor. 10.50

——— Traité élémentaire de Statique. 8vo. Hf. mor. . . . 1.75

MORIN. Aide-mémoire de Mécanique Pratique. 8vo. 2.00

MONTFERRIER. Encyclopédie Mathématique ou Exposition complète de Toutes les Branches des Mathématiques d'apres les Principles de la Philosophie des Mathématiques de Hoéné Wronski. 3 vols. 8vo. To be continued in monthly Livraisons. 6.00

NAVIER. (Ketten) Hängbrüchen. 4to. 5.00

OLIVIER. Théorie Géométrique des Engrenages. 4to. Hf. mor. . 3.75

——— Developpements de Géométrie Descriptive. 2 vols. 4to. Hf. mor. 9.00

——— Complément de Géométrie Descriptive. 2 vols. 4to. 9.00

——— Mémoires de Géométrie Descriptive, Theorique, et Appliquée. 2 vols. 4to. Hf. mor. 9.00

——— Application de la Géométrie Descriptive. 2 vols. 4to. 10.00

——— Cours de Géometrie Descriptive. 2 vols. 4to. Hf. mor. 10.00

POINSOT. Eléments de Statique. 8vo. Hf. cf. 2.50

POISSON. Mécanique. 2 vols. 8vo. 6.00

PONCELET et LESBROS. Experiences Hydrauliques. 4to. Hf. mor. 4.00

PONTECOULANT. Système du Monde. 4 vols. 8vo. Hf. cf. . 16.00

POUILLET. Eléments de Physique Experimentale. 3 vols. 8vo. Hf. cf. 7.50

SERRET. Cours d'Algèbre supérieure. 8vo. Hf. cf. 3.50

——— Eléments d'Arithmétique. 8vo. Hf. cf. 1.50

TERQUEM. Nouvelles Annales de Mathématiques. Monthly. Per annum 4.00

——— Nouvelles Annales de Mathématiques, 1856–7. 2 vols. Hf. cf. 6.50

——— Nouvelles Annales de Mathématiques. Tomes VIII. à XVII. Corresponding to the Years 1849–1859. 10 vols. Hf. cf.

——— Bulletin Bibliographie, d'Histoire, et de Biographie Mathématiques. 8vo. Hf. cf. 3.00

TRESCA. Géométrie Descriptive. 8vo. Hf. mor. 3.00

VERHULST. Traité élémentaire des Fonctions Elliptiques. 8vo. Hf. cf. 3.50

VIEILLE. Cours complémentaire d'Analyse et de Mécanique. 8vo Hf. cf. 2.50

SCIENTIFIC BOOKS.

AIRY. Mathematical Tracts. 4th ed. $4.00
ARAGO. Astronomie. 4 vols. 8vo. hf. mor. 12.00
——— Notices Biographiques. 3 vols. 8vo. hf. mor. . . . 9.00
——— Notices Scientifiques. 4 vols. 8vo. hf. mor. . . . 12.00
——— Mémoires Scientifiques. Vol. I. 8vo. hf. mor. . . . 3.00
——— Voyages. 8vo. hf. mor. 3.00
BOOLE. Calculus of Finite Differences 3.00
BOW. Treatise on Bracing. 8vo. 1.12
BROUGHAM (Lord). Tracts, Mathematical and Physical . . . 2.25
BUCK. Oblique Arches of Bridges. 4to. 3.75
Builder (The). A Monthly Magazine for Architects. Per annum . 8.00
Cambridge Problems with Solutions, 1860, Watson and Routh . . 2.25
Civil Engineer and Architect's Journal. Monthly. 4to. London. Per annum, post-paid 7.50
CLARK. Britannia and Conway Tubular Bridges. 2 vols. 8vo, and atlas folio 28.50
DE LA RIVE. Traité d'Electricité théorique et appliquée. 3 vols. 8vo. hf. mor. 9.00
——— Treatise on Electricity, in Theory and Practice. 3 vols. 8vo. 22.00
EMY. Traité de la Charpenterie. 2 vols. in 4to, et atlas in fol. hf. mor. 25.00
FERGUSON. Handbook of Architecture. 2 vols. 8vo. hf. cf. . . 12.00
Lady and Gentleman's Diary, Poetical and Mathematical, for 1860 and 1861 . . . . . Each $0.40
Mathematician (The). Edited by Davies, Rutherford, and Fenwick. 3 vols. 8vo. 9.00
Also Vols. II. and III. separately 4.00
MAXWELL, J. C. On the Stability of the Motion of Saturn's Rings. 4to. paper 1.80
Nautical Almanac (English) for 1864 . . . . . 1.00
POTTER. Physical Optics. 2 parts . . . . . Each 2.25
———. Treatise on Hydrostatics 2.25
PRATT. Theory of the Attraction of the Earth 2.00
PUCKLE. Conic Sections 2.00
REGNAULT. Manuel des Aspirantes au Grade d'Ingénieur des Ponts et Chaussées. Partie théorique et Partie pratique. 4 vols. 8vo. hf. cf. 10.00
RŒBLING. Memoir of the Niagara Falls and International Suspension Bridge. 8vo. 7.50
TODHUNTER. Algebra 2.25
——— Plane Trigonometry 1.50
——— Spherical Trigonometry 1.25
——— Analytical Geometry, Examples of 1.25
——— Conic Sections 3.00

SECOND-HAND MATHEMATICAL BOOKS.

AIRY. Trigonometry. Tides and Waves. Figure of Earth. 4to. $4.50
BAGAY. Nouvelles Tables Astronomiques et Hydrographiques. 4to. hf. cf. *Scarce* 10.00
BLAND. Algebraical Problems. 8vo. 1.75
——— Key to do. 8vo. 1.75
——— Philosophical Problems. 8vo. 1.50
——— Geometrical Problems. 8vo. 1.75
——— Elements of Hydrostatics 1.50
DE MORGAN. Essay on Probabilities 1.00
——— The Globes, Celestial and Terrestrial 1.25
Diarian Miscellany, Mathematical and Poetical, by Hutton. 5 vols. 12mo. cf. 4.00
Cambridge Problems. From 1801 to 1820. 8vo. 1.00
EARNSHAW. Dynamics. 8vo. 2.50
GRANT. Plane Astronomy. 8vo. 1.25
GREGORY AND WALTON. Application of Analysis to Solid Geometry. 8vo. 3.00
GREGORY. Differential and Integral Calculus, by Walton. 8vo. 2d ed. 4.50
HERSCHEL. Calculus of Finite Differences. 8vo. 4.00
HYMER. Conic Sections. 3d ed. 8vo. 2.00
——— Analytical Geometry of Three Dimensions. 1st ed. 8vo. 1.00
——— do. do. 2d ed. 8vo. 1.50
——— Plane and Spherical Trigonometry. 2d ed. 8vo. . . 1.25
——— do. do. 3d ed. 8vo. . . 1.50
——— Integral Calculus. 3d ed. 8vo. hf. cf. 4.00
HYMER. Astronomy. 2d ed. 8vo. $2.00
LACROIX. Elements of Algebra, translated by Prof. John Farrar. 5th ed. 8vo. 1.50
Ladies' Diary, Poetical and Mathematical. 1784-1790. 16mo. . 1.00
LAPLACE. Elementary Illustrations of Celestial Mechanics. 8vo. . 1.25
——— Analytical Mechanics, Translated by Rev. John Tophs 1.00
LARDNER. System of Algebraic Geometry. 8vo. 1.75
——— Elements of Euclid. 5th Edition. 8vo. 1.50
——— Differential and Integral Calculus. 8vo. 2.00
LEYBOURN. Mathematical Questions with Solutions. 4 vols. 8vo. 8.00
——— Mathematical Institutions. 4to. 1.50
Mathematical Register; or, Diarian Repository. 4to. 2.50
Mathematical Companion, The Gentleman's, for the Years 1810-11-12-14-15-17-18-19-20-24-25. 16mo. Unbound. 11 Nos. . 2.00
MOIGNO. Leçons de Calcul Differential. Vol. I. Hf. cf. Scarce. 9.00
MOSELEY. Hydrostatics and Hydrodynamics. 8vo. Scarce. . . 4.00
O'BRIEN. Mathematical Tracts. Part I. 0.75
PEACOCK. Algebra 2.00
POISSON. Traité de Mécanique. 2 vols. 8vo. hf. bd. 5.00
PRICE. Differential Calculus. Vol. I. 1st ed. 1.50
PUCKLE. Conic Sections. 1st ed. 12mo. 1.50
STEVENSON. Algebraic Equations. 8vo. 1.00
TODHUNTER. Analytical Statics. 12mo. 2.00
WAUD. Algebraic Geometry. 8vo. Hf. cf. 1.50
YOUNG. Algebraical Equations. 12mo. 1.00

☞ Orders for the Importation of English, French, and German Books will be promptly executed, and at moderate prices. Early copies of all the new mathematical books published in England and France will be received as soon as published.

BOOKS FOR PUBLIC LIBRARIES IMPORTED FREE OF DUTY.

TEXT-BOOKS USED IN HARVARD COLLEGE.

THE ORATION OF ÆSCHINES AGAINST CTESIPHON. With Notes, by PROFESSOR J. T. CHAMPLIN. 12mo. cloth. $0.88
THE CLOUDS OF ARISTOPHANES. With Notes, by PROFESSOR C. C. FELTON. 12mo. Cloth 1.00
THE BIRDS OF ARISTOPHANES. With Notes, by PROFESSOR C. C. FELTON. 12mo. cloth 1.00
THE PANEGYRICUS OF ISOCRATES. With Notes, by PROFESSOR C. C. FELTON. 12mo. Cloth 75
SELECTIONS FROM THE GREEK HISTORIANS. With Notes, by PROFESSOR C. C. FELTON. 12mo. half mor. . . . 1.50
SELECTIONS FROM MODERN GREEK WRITERS, IN PROSE AND POETRY. With Notes, by PROFESSOR C. C. FELTON. 12mo. Cloth 1.00
THE AIAS OF SOPHOKLES. With Critical and Explanatory Notes, by J. B. M. GRAY. 12mo. cloth 1.25
PRONUNCIATION AND HISTORY OF THE GREEK ALPHABET. By E. A. SOPHOCLES, Professor in Harvard College. 12mo. Cloth 1.00
CICERO. BRUTUS. Edited, with Notes, by PROFESSOR C. BECK. 16mo. Cloth 75
CICERO. Immortality of the Soul; The Tusculan Disputations, Book First; The Dream of Scipio; and Extracts from the Dialogues on Old Age and Friendship. With English Notes, by THOMAS CHASE, Tutor in Harvard College. 16mo. Cloth. . .75
CICERO. PRO CLUENTIO. With English Notes, by AUSTIN STICKNEY, M. A., Professor of Latin in Trinity College. 12mo. Cloth 75
CONSTITUTIONAL DOCUMENTS OF ENGLAND AND AMERICA, from Magna Charta to the Federal Constitution of 1789. Compiled and Revised, with Notes, by F. BOWEN, Professor of Moral Philosophy in Harvard College. 8vo. Cloth . . . $0.88
HODGES, R. M. PRACTICAL DISSECTIONS. By RICHARD M. HODGES, M. D., Demonstrator of Anatomy in the Medical Department of Harvard College. 16mo. 1.00
HORACE. With Notes, by A. J. MACLEANE, M. A. Revised and Edited by R. H. CHASE, A. M. With Introduction to the Metres, by PROFESSOR CHARLES BECK. Revised edition. 12mo. half mor. 1.25
LATHAM'S ELEMENTARY ENGLISH GRAMMAR. Revised by PROFESSOR CHILD. With an Appendix, by PRESIDENT GOODWIN, Trinity College. 16mo. Cloth 75
PEIRCE, J. M. ANALYTIC GEOMETRY. 8vo. 1.50
ROELKER, B. A GERMAN READER FOR BEGINNERS. Compiled by BERNARD ROELKER, A. M., Instructor in Harvard University. 12mo. Cloth 1.00
SYNTAX OF THE MOODS AND TENSES OF THE GREEK VERB. By W. W. GOODWIN, Ph. D., Prof. in Harvard College. 12mo. Cloth 1.25
VERNON, EDWARD JOHNSTON, B. A. GUIDE TO THE ANGLO-SAXON TONGUE. 12mo. Cloth. 1.25
WHATELY. LESSONS ON MORALS AND CHRISTIAN EVIDENCES. Edited by REV. F. D. HUNTINGTON. 16mo. . . .75

PUBLISHED AND FOR SALE BY

SEVER & FRANCIS,

Booksellers to the University, Cambridge.

THE NEW AMERICAN CYCLOPÆDIA.

D. APPLETON & CO.,

Nos. 443 & 445 Broadway, New York,

HAVE JUST PUBLISHED

Vol. XI. of the

New American Cyclopædia,

A POPULAR DICTIONARY OF GENERAL KNOWLEDGE,

EDITED BY

GEORGE RIPLEY and CHARLES A. DANA,

Aided by a Numerous Select Corps of Writers from all parts of the United States.

This work is being published in about 16 large octavo volumes, each containing 750 two-column pages.

VOLS. I. TO XI. NOW READY.

To be followed by a new Volume every three months.

Price — In cloth, $3; sheep, $3.50; half morocco, $4; half Russia, $4.50 each.

From De Bow's Review, New Orleans.

"This great work, published by the Appletons of New York, has reached its eighth volume. It is one of the most valuable Cyclopædias in the world, and for Americans the most valuable. It is contributed to by many of the ablest and best writers of the South, and is as fair to every section of the Union as it is possible for a work to be. It will be completed in about seven other volumes."

The following notice in *Silliman's American Journal of Sciences and Arts*, for March, is from the pen of the venerable senior editor, one of the highest living authorities on the subject: —

"In the American Cyclopædia the articles on Science are numerous and valuable, and elevate the work to the character of a compendium of modern science. These articles are in most cases written with decided ability, and evidently by persons who are familiar with the topics they discuss. While many of the less important subjects are presented with luminous brevity, others are more fully expanded. Among these are many topics of Natural History. Chemistry is presented with its equivalence and laws of combination, illustrated by many of its modern discoveries and practical applications; of the latter, an example is found in the full account of the manufacture of gelatine, of beer, and bricks, and in the ample history of gas-lighting. Neology, voltaic electricity, magnetism, and other departments of pure or applied science, are treated with reasonable fulness. As a literary work the Cyclopædia is written in a pure and chaste style, and exhibits the candor and fairness which should ever adorn a record of universal knowledge. L. S."

OUR SUBSCRIPTION LIST

Already numbers TWELVE THOUSAND names, from every section of the country, which evidences the universal popularity of this NATIONAL WORK.

THE NEW AMERICAN CYCLOPÆDIA

Is sold exclusively by subscription, and Agents have been appointed for almost all sections of the Union. In case, however, Agents are not to be found, we will receive subscriptions and forward copies of the work, free of expense, on receipt of the price. Persons at a distance may send money for one or more volumes at a time, it not being necessary to take the whole eight volumes at once, unless they feel inclined to do so. Those living in the city will find a subscription-book at our store, where their names will be entered, and the volumes, as published, sent to any part of the city.

WARREN'S

Descriptive Geometry.

ORTHOGRAPHIC PROJECTIONS.

BOOK I. — General Problems.

BOOK II. — Applications to Isometrical Projections, Spherical Trigonometry, Topographical Projections, and Graphic Transformations.

8vo. 447 pp., and 36 sheets of plates. Price, $3.00.

BY S. EDWARD WARREN, C. E.,

PROFESSOR OF DESCRIPTIVE GEOMETRY AND GEOMETRICAL DRAWING IN THE RENSSELAER POLYTECHNIC INSTITUTE, TROY, N. Y.

This work has been prepared as the result of a number of years' experience in teaching the interesting and useful branch of mathematical science of which it treats, in the Rensselaer Polytechnic Institute.

It has been warmly welcomed and commended in discriminating and independent critical notices, some of which are appended.

The study of Descriptive Geometry, as a *means of intellectual training*, as well as an *aid in important industrial pursuits*, has peculiar merits, and the attention of professors, teachers, and students of Pure or Applied Mathematics in the scientific schools, colleges, &c., and members of the Engineering profession, are invited to an examination of the above work.

RECOMMENDATIONS.

"Professor Warren's book is far more comprehensive than any work on Descriptive Geometry before offered to American, or, so far as we know, to English readers. . . . It has many positive merits." — *Mathematical Monthly, February*, 1861.

From WM. MITCHELL GILLESPIE, LL. D., *Prof. Civil Engineering, Union College, N. Y.*

"I have looked over your work on Descriptive Geometry with surprise and delight, — surprise that the most comprehensive book on that subject in the English language should be an American one, and delight that it should be so good a one, &c., &c. I shall recommend it to my mathemat ical friends."

From J. S. WOODMAN, *Prof. Civil Engineering, Dartmouth College, N. H.*

"I have hastily examined your excellent treatise on the 'Orthographic Projections of Descriptive Geometry,' and like it much. The parts of Descriptive Geometry you have undertaken are so systematized and so fully discussed, that your book must have an important influence in bringing that science into public notice and favor."

JOHN WILEY, Publisher and Importer,

56 WALKER STREET, NEW YORK.

☞ In press, by the same author, "A Manual of Elementary Geometrical Drawing, involving three Dimensions." With numerous plates. 1 vol., 12mo. Price, $1.25.

RECENT IMPORTANT WORKS.

Annual of Scientific Discovery for 1861; or, YEAR-BOOK OF FACTS IN SCIENCE AND ART, exhibiting the most important Discoveries and Improvements in Mechanics, Useful Arts, Natural Philosophy, Chemistry, Astronomy, Meteorology, Zoölogy, Botany, Mineralogy, Geology, Geography, Antiquities, &c. Edited by David A. Wells, A. M. 12mo. Cloth, $ 1.25. (*Just published.*)

No other single publication, American or foreign, presents such a mass of fresh, varied, interesting, and valuable information, adapted to the wants of persons in almost every walk of life; and its acquisition has become a necessity to all who would acquaint themselves with the progress of discovery with each successive year.

The Romance of Natural History. By Philip Henry Gosse. With numerous Elegant Illustrations. 12mo. Cloth, $ 1.25. (*Just published.*)

One of the most attractive issues of the season. Its charming descriptions and admirable illustrations are giving it a marked popularity. The first edition was exhausted in a few days.

The Pulpit of the American Revolution; Or, The Political Sermons of the Period of 1776. With a HISTORICAL INTRODUCTION, NOTES, and ILLUSTRATIONS. By J. Wingate Thornton, A. M. 12mo. Cloth, $ 1.25. (*Just published.*)

"This work contains the sum and substance of the whole history of the American Revolution."

The Puritans; or the Court, Church, and Parliament of England, during the reigns of Edward VI. and Elizabeth. By Samuel Hopkins, Author of "Lessons at the Cross," etc. In 3 vols. Vols. I. and II. *now ready.* 8vo. Cloth, $ 2.50 per vol.

This is a masterly work, by one of the most distinguished of living authors.

Essays in Biography and Criticism. By Peter Bayne, M. A., Author of "The Christian Life, Social and Individual." Arranged in two Series or Parts. 12mo. Cloth, $ 1.25 each.

"This is a book of rare literary excellence. Its analytical powers, its earnestness of purpose, and its wealth of language, are alike remarkable." —*Mrs. Sigourney.*

The Christian Life; SOCIAL AND INDIVIDUAL. By Peter Bayne, M. A. 12mo. Cloth, $ 1.25.

"It is full of noble thought and brilliant illustration." — *Hon. R. C. Winthrop.*

Thesaurus of English Words and Phrases; so classified and arranged as to facilitate the expression of ideas, and assist in literary composition. By Peter Mark Roget, late Secretary of the Royal Society, London, etc. Revised and edited, with a List of Foreign Words defined in English, and other additions, by Barnas Sears, D. D., President of Brown University. A New American Edition, with Additions and Improvements. 12mo. Cloth, $ 1.50.

"A work of great utility. It will give a writer the word he wants, when that word is on the tip of his tongue, but altogether beyond his reach." — *N. Y. Times.*

British Novelists and their Styles. Being a Critical Sketch of the History of British Prose Fiction. By David Masson, M. A., author of "The Life and Times of John Milton," etc., etc. 16mo. Cloth, 75 cents.

"This book is a hearty, honest, and learned history of the British novel,—remarkable for its acute and philosophical criticism, and the classic purity and elegance of its style." — *Providence Journal.*

The Preacher and the King; or, Bourdaloue at the Court of Louis XIV. By L. F. Bungener. With a Likeness and Biographical Sketch of the Author. 12mo. Cloth, $ 1.25.

The Priest and the Huguenot; or, Persecution in the Age of Louis XV. By L. F. Bungener, Author of "The Preacher and the King." 2 vols. 12mo. Cloth, $ 2.25.

Plurality of Worlds. With an Introduction by Edward Hitchcock, D. D., and a Supplementary Dialogue, in which the author's reviewers are reviewed. 12mo. Cloth, $ 1.

☞ The above works are worthy of special attention. Bungener is a writer of great originality and power; and "The Plurality of Worlds" is one of the marked productions of modern times.

Geographical Studies. Translated from the German of Carl Ritter. By Rev. W. L. Gage. With a Sketch of the Author's Life and a Portrait. 12mo. Cloth, $ 1. (*Ready in a few days.*)

This volume contains the grand generalizations of Ritter's Erdkünde, his Lectures on the relations of Geography and History, and important papers on Physical Geography.

Mental Philosophy; Including the Intellect, Sensibilities, and Will. By Joseph Haven, Professor of Intellectual and Moral Philosophy, Amherst College. Royal 12mo. Cloth, $ 1.50.

Prof. Park, of Andover, says: "It is DISTINGUISHED for its clearness of style, perspicuity of method, candor of spirit, acumen, and comprehensiveness of thought. I have been heartily interested in it."

"Earnestness, candor, and accuracy and elegance of diction characterize every page, and render the work as interesting to the general reader as it is valuable to the scholar." — D. H. COCHRAN, *Principal and Prof. of Moral and Intellectual Philosophy, State Normal School, Albany.*

This work has been extensively adopted as a text-book in Colleges and High Schools throughout the country.

Moral Philosophy; Including Theoretical and Practical Ethics. By Joseph Haven, D. D., late Professor of Moral and Intellectual Philosophy in Amherst College; Author of "Mental Philosophy." Royal 12mo. Cloth, $ 1.25.

It is scientific in method and thorough in discussion, and its views on unsettled questions in morals are discriminating and sound. It treats largely of Political ethics, — a department of morals of great importance to American youth, but generally overlooked in text-books. In the history of ethical opinions, it is unusually rich and elaborate.

Lectures on Logic. By Sir William Hamilton, Bart., Professor of Logic and Metaphysics in the University of Edinburgh. With Notes from original materials, and an Appendix containing the latest development of his new Logical Theory. Edited by H. Longueville Mansel, B. D., Oxford, and John Veitch, A. M., Edinburgh. Royal 8vo. Cloth, $ 3.00.

This volume completes the Metaphysical and Logical series of Sir William Hamilton, and will enable American scholars to possess themselves of the invaluable works of this eminent author, in two elegant octavo volumes, at six dollars, — half the price of the English edition.

Lectures on Metaphysics. By Sir William Hamilton, Bart. Edited by Prof. H. L. Mansel, B. D., Oxford, and John Veitch, M. A., Edinburgh. Royal 8vo. Cloth, $ 3.00.

Hamilton's Lectures on Metaphysics have been received with eminent favor, and already adopted as a text-book in Harvard, Yale, Princeton, and other distinguished Colleges.

Elements of Moral Science. By Francis Wayland, D. D., late President of Brown University. 12mo. Cloth, $ 1.25.

Elements of Political Economy. By Francis Wayland, D. D., 12mo. Cloth, $ 1.25.

The above works by Dr. Wayland are used as text-books in many of the colleges and higher schools throughout the Union, and are highly approved.

The Elements of Geology; Adapted to Schools and Colleges. With numerous Illustrations. By J. R. Loomis, President of Lewisburg University, Pa. 12mo. Cloth, 75 cents.

"It is surpassed by no work before the American public." — M. B. ANDERSON, LL. D., *President of Rochester University.*

Principles of Zoology; Touching the Structure, Development, Distribution, and Natural Arrangement of the RACES OF ANIMALS, Living and Extinct; with numerous Illustrations. For the use of Schools and Colleges. Part I. COMPARATIVE PHYSIOLOGY. By Louis Agassiz and Augustus A. Gould. Revised edition. $ 1.00.

The Earth and Man; Lectures on Comparative Physical Geography, in its relation to the History of Mankind. By Arnold Guyot. With Illustrations. 12mo. Cloth, $ 1.25.

Prof. Louis Agassiz, of Harvard University, says: "It will not only render the study of Geography more attractive, but actually show it in its true light."

Hugh Miller's Works. Seven volumes, uniform style, in an elegant box. Embossed Cloth, $ 8.25.

GOULD & LINCOLN, BOSTON.

PROFESSOR WOOD'S NEW BOTANY,

New Edition, Enlarged.

Teachers, pupils, and amateurs will find in the new Class-Book of Botany the following peculiar advantages: —

First. The Scientific Treatise, embraced in Parts I., II., and III., is remarkable for its comprehensiveness, clearness, and brevity; is divided into short paragraphs, suited to the learner's convenience, and each paragraph, with the topic prefixed in capitals, is adapted to recitation and reviews.

Second. The Flora comprehends a wider territory than that of any school Botany now in use, extending from the Atlantic to the Mississippi, and from the St. Lawrence to the Gulf of Mexico.

Third. The Flora comprehends not only our native, spontaneous vegetation, but also one thousand species of cultivated plants, — almost our entire exotic flora, growing in the field, the garden, and the conservatory; thus rendering the study of Botany as practicable in the city as in the country.

Fourth. The descriptions of species are unusually full, graphic, and popular in style.

Fifth. The Tables for analysis are far in advance of those in the former editions, both in simplicity and extent, reaching now from the Grand Division to the Species, and by their peculiar form adapted to *class-exercise* throughout the entire route; thus rendering the pursuit at once a vigorous discipline and an exciting amusement.

Sixth. The entire course of the study is included in one volume. Price $2.00.

PUBLISHED BY A. S. BARNES & BURR,

51 & 53 John Street, New York,

Publishers of the National Series of Standard School Books.

A New French Book,

JUST PUBLISHED.

A New French Course. — Specimen-Pages of Pujol and Van Norman's Complete French Class-Book, designed for Seminaries and Academies, also for Private Students.

NEW YORK: A. S. BARNES & BURR.

After a careful examination of the specimen-pages of the new French course, by Messrs. Pujol and Van Norman, we feel justified in calling the attention of all teachers and students of French to this work as one promising to be of great importance. It is not always safe to judge beforehand of the value of a forthcoming work by a few specimen-pages; but there is so much excellency in these few pages, and so many proofs of a real improvement over former works of the kind, that we have no doubt whatever that the book will meet with a favorable reception, and an extensive introduction as a text-book. The work will contain, in one volume, a complete system of theoretical and practical instruction. It is divided into four parts: 1. A theoretical and practical grammar, on a plan entirely new, with exercises in both French and English, and a series of vocabularies, forming an adequate dictionary. 2. A complete and orderly development of French syntax, with exercises in both French and English, and polite conversations, illustrative of the idiomatical rules. 3 A new system of teaching French conversation. 4. Progressive lessons in translating, and selections from the French classics, which are headed with biographical sketches of the authors. An appendix of polite and commercial correspondence will complete the work. It is, of course, principally the first part of the book which will decide the usefulness of the work. The specimens contained in the pamphlet strike us as decidedly superior to any other French book we are acquainted with. Some points which especially attracted our attention are: the brevity and clearness of the grammatical rules; the judicious selection of the sentences for grammatical exercises; the equal length of the lessons; the skilful use made of different kinds of types in order to call the attention of the pupil to the rules of the lesson. Every page we have examined gives ample evidence that the entire system has been practically tried by the authors before writing it down. We are satisfied that the book will be a model text-book, the peculiar features of which no teacher ought to omit to study.

A. S. BARNES & BURR'S

Recent Publications.

TEACHERS OF VERMONT AND NEW ENGLAND,

DO YOU WANT THE BEST BOOKS?

THEN SEND TO A. S. BARNES & BURR, 51 & 53 JOHN STREET, NEW YORK,

FOR THEIR

List of Valuable Text-Books.

AMONG THEIR LAST ISSUES ARE:

1. **Professor Alphonso Wood's New Edition of the Class-Book of Botany.**

   Designed for all parts of the Union. The most thorough and accurate Text-Book ever issued from the American press. 840 pages. Price, $2.00. (The old Edition is still published, price $1.50.)

2. **Pujol & Van Norman's complete French Class-Book.**

   Embracing *Grammar*, *Conversation*, and *Literature*, with an *adequate Dictionary*, all in one volume. Price, $1.50. No French book extant will compare with this excellent treatise for giving a thorough and systematic knowledge of the French Language.

3. **Mansfield's Political Manual.**

   Being a complete View of the Theory and Practice of the General and State Governments of the United States, — adapted to the use of Colleges, Academies, and Schools. Price, 75 cents. This work is exactly suited to the times, when the people of the nation should understand the principles of the Government in which they live.

4. **Mrs. Willard's School Astronomy and Astronomical Geography.** Price, 75 cents.

5. **Mrs. Willard's last Edition of her excellent History of the United States.**

   This book is brought down to the John Brown raid. Price, 75 cents.

6. **Zachos's Analytic Elocution.**

   Being an Analysis of the Powers of the Voice, for the purpose of Expression in Speaking. Illustrated by copious examples, and marked by a system of Notation. By J. C. Zachos, A. M. Price, $1.00.

7. **Peck's Ganot's Popular Physics.**

   Being an Introductory Course of Natural Philosophy, with over 300 splendid Illustrations. Price, $1.00.

BALTIMORE, *January* 28, 1861.

Messrs. A. S. BARNES & BURR:

Gentlemen, — Permit me to acknowledge the receipt of a copy of "Peck's Ganot." After an examination from beginning to end, I do not hesitate to pronounce it one of the best (if not the very best) Essays upon the subject, now before the public. I am delighted with it, and shall spare no pains to have it introduced wherever my influence may be sufficient to accomplish so desirable an object.

Very respectfully yours,

R. N. WRIGHT,

Principal of Central High School.